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LECTURE SERIES

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By

JEROME ROTHENBERG

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Opinions or value judgements expressed in this book are those of the author and do not necessarily represent those of the Center of Planning and Economic Research

CENTER OF PLANNING AND ECONOMIC RESEARCH

The Center of Planning and Economic Research was established in 1961 as an autonomous public organization. It was originally called the Center of Economic Research, and its functions included basic research in the structure, behaviour and problems of development of the Greek economy, as well as the advanced training of young Greek economists. For the establishment and operation of the Center, substantial financial assistance was provided by the Ford and Rockefeller Foundations and the United States Mission to Greece.

In 1964, the Center of Economic Research was reorganised as the Center of Planning and Economic Research. In addition to its function as a research and training institute, the Center was assigned the following tasks by the State: (1) *the preparation of long-term plans for economic development*; (2) *the evaluation of public investment programmes and*; (3) *the study of short-term developments in the Greek economy and advice on current problems of economic policy. Studies of important problems of the Greek economy and of fundamental policy issues are published in a Research Monograph Series.*

The Center has developed a successful scholarship programme. In collaboration with foreign universities and other organizations, a number of the Center's young economists are sent abroad for post-graduate study. In addition, the Center organizes an annual series of seminars and lectures for the benefit of its own staff, and other economists and scholars. Many of these activities have been conducted by distinguished foreign scholars invited by the Center. Papers and lectures presented at *these seminars are published in the Center's Training Seminar Series and Lecture Series. Finally, the Center has established and maintains close relations with similar institutions throughout the world, communicating with them regularly on particular problems and research techniques.*

From 1961 to 1966 the Center was assisted by the University of California at Berkeley. The latter helped in the selection of foreign scholars who joined the Center on an annual basis, as well as in an exchange programme including visits of American students to the Center and the enrollment of Center staff members as post-graduate students at American Universities.

In 1966 an agreement was concluded with Harvard University. With financial assistance provided by the Ford Foundation, Harvard's Development Advisory Service is providing the Center with the services of foreign experts, who advise the Center in its research. planning and educational activities. The agreement also provides for a limited number of scholarships, to be used by members of the Center staff for post-graduate studies abroad.

CONTENTS

The subject matter of this book is based on lectures delivered at the Center of Planning and Economic Research in May 1966, while the author was on leave from Northwestern University. An earlier draught was prepared while at the Rand Corporation, Santa Monica, California.

I

INTERGENERATIONAL TRANSFERS AND WELFARE COMPARABILITY

A. WELFARE COMPARISONS OVER TIME

The problem of welfare comparisons over considerable time intervals is simply that the populations in the two periods being compared are not the same. The normative criteria economists use for making welfare comparisons generally require an unchanging population. Indeed, the most frequently employed Pareto criterion requires that the welfare of each and every individual in both situations be determined, and that such person by person determination be a *necessary* condition for any comparison to be made. Information about the welfare of any individual can neither be substituted for, nor offset, by information about some other individual's welfare. Criteria other than the Pareto criterion are less demanding in this regard. They allow welfare changes for some individuals to be *offset* by welfare changes for other individuals. In other words, they provide for interpersonal comparisons of welfare — a step toward interchangeability of persons within a population. But this interchanges not individuals, but individuals' welfare changes between the two situations being compared: each individual must be present in both situations in order that his welfare change be determined. In the situations we intend to examine, the key feature is that some — if not all — individuals are present in only one of the two situations. In its extreme form, the problem concerns choice situations where the welfare of one «generation» is compared with that of another «generation»: i.e., where there is presumably no overlap at all in the populations being compared.

A solution to the problem would seem to require that some linkage with welfare significance be found between individuals at different time periods. This paper has as its first task to examine some possible linkages and to develop their implications for intertemporal welfare comparisons.

B. THE PROCESS OF POPULATION CHANGE: THREE MODELS

The two single keys on which our treatment of linkage is based are: (1) the population composition changes almost continuously, with one stream of members dropping out, and another stream being added; (2) the new additions to the population are familially linked to the existing members. This section will be devoted to the first. The next section will deal with the second.

The near-continuity of compositional changes means that there is literally no sharp demarcation between generations. Unless one has in mind comparing periods so separated as to have no individuals in common, the use of the term «generation» is figurative, not literal. At any one time, the population will contain the «present generation» and the «past generation» and the «future generation», as well as generations in between. Any decision taken at that time which affects the future will affect all these, and furthermore, will be made by members or representatives of all.

When only small intervals are involved, the populations at two different dates have preponderant overlap. Moreover, for those in the overlap either they themselves or their representatives helped make the decisions in the earlier time that helped shape the later. When long intervals are involved, so that overlap is nearly or totally absent, we can search for comparability in terms of the intermediate periods between them which do have substantial pairwise overlap. Unlike price index theory, where a set of intermediate price relatives between two distant dates do not suffice to persuade that the distant pair is comparable if quantity weights, commodity qualities, etc., differ greatly between them, the linkage of adjoining relatives here does carry persuasive power, so long as some continuing control over intertemporal resource allocation inheres in the intervening periods.

The linkage we seek between different evaluative groups over time will depend on this existence of a nondegenerate age distribution of the population at each moment of time, and its implication of near-continuous generation of future generations by and alongside present and past generations. This relationship clearly does not suffice, however, to create the full linkage we seek. We must examine the nature of the taste changes that occur as population composition changes over time.

A basic problem is: what happens to the tastes (preferences) of each individual as he passes through his ageing cycle over time? The answer is not clear-cut, because while empirical facts about wants, tastes, preferences, styles of life are known, the normative interpretation of these facts is not at all unambiguous. It is known, for example, that an individual typically retains a personality core, a basic configuration of attitudes and values, throughout most of his life. On the other hand, he experiences changes in tastes about specific commodities: (1) as conditions of his life change, (2) as new commodities are introduced, (3) simply as a result of accumulating experience with different types of consumption. In particular, his style of life, and with it, his commodity preferences, change appreciably as he goes through different basic stages of life. In terms of the *normative comparability* of decisions made by him at different times — which is itself a normative decision — which of these shall be stressed: the constancy or the variability? The existence of just any taste changes does not suffice to decide in favour of the latter, since the preferences to be considered constant for any individual could be formulated as a life-stage sequence of commodity comparisons, rather than simply as preferences about present consumption. So the answer is not obvious. Let us consider three possible treatments.

Model 1

Each individual remains unchanged through life. Then the comparability of temporally different populations is simply the difference in the sheer identity of members. Populations become different evaluative machines insofar as the composition changes. The decision-making unit is the individual.

Model 2

Each individual assumes the tastes of his socio-economic-age group. Since the array of such groups continues over time even as their membership composition changes, aggregate tastes change only as the socio-economic-age distribution or total size of the population changes. A constant distribution among these categories, despite population births and deaths, would keep the group's tastes unchanged. The decision-making unit is the age class.

Model 3

Each individual changes over time, but in ways more idiosyncratic than under Model 2. The same individual, in two different periods where his tastes differ, would be treated as two different individuals. The decision-making unit is the individual preference scale.

Models 1 and 2 involve the least problem of evaluative comparability over time. Indeed, Model 2 largely dispenses with interpersonal comparability as any kind of evaluative problem by essentially defining away individual uniqueness. Tastes are group phenomena, and individuals within the group are interchangeable. Both models differ from Model 3 in assuming a critical degree of regularity in taste patterns over time — Model 1 assuming that the individual's present projection of the unique course of his whole future sequence of life styles represents accurate prediction; Model 2 assuming that the dependable regularity is better explained by the individual's socio-economic-age group membership than by any idiosyncratic present perspective. Model 3 assumes that important changes in tastes neither fit the individual's present orientation nor his age group. Regularities of both sorts are, as has been indicated, observable. But other changes are noted as well, many having to do with the introduction of new commodities. These lessen the overall regularities which lend support to Models 1 and 2. The result is that observable experience does not lend unqualified support to any of the three treatments.

Choice among the three models requires the making of a *normative*, not empirical, judgement. The normative selection among them, like the empirical, is hardly likely to be definitive. In the present instance we shall make a conservative choice. Models 2 and 3 are both extreme positions. Model 2 essentially wipes out individual uniqueness, substantially oversimplifying the normative problem. Model 3 wipes out any stability over time, thereby hopelessly complicating the normative problem. We shall tentatively work with Model 1 therefore, not as a definitive choice, but only in the absence of more strongly persuasive evidence that a more extreme position is preferable.

Under our chosen model, then, the individual is deemed to possess a long-term, «lifetime» utility function, one that takes account of the manner in which changing age and accumulating experience with commodities can affect his preferences among particular commodity bundles. It evaluates different long-run strategies by which he exposes himself to various broad consumption styles over time, each prospectively attempting to influence the temporal pattern of his evolving short-run preferences. (This distinction between long- and short-run preferences will be significant below).

C. PARENTS AND HEIRS

So far we have argued that populations change little by little over time, overlapping appreciably between any two temporally close periods. Moreover, every two populations widely separated in time can be linked by introducing a sequence of intervening populations temporally close to one another. Within each population we have assumed that each individual remains essentially the same decision-making unit throughout his life-time. We now attempt to provide a closer linkage for the piecemeal changes that occur in the population, a linkage that will sustain normative comparability even between populations that overlap considerably. The linkage is in the parent-heir relationship (P-H).

Consider two individuals A and A′, such that A′ is the heir of A. Let us assume, without doing too much violence to reality, that every bequeather is the parent of at least one heir. So A is father to A′. Now, we argue that the decisions A makes about resource allocation affecting the period after his own death are taken *on behalf of* his heir(s) A′. This is of real importance. This interpersonal linkage enables us to deal with the problem of interpersonal comparability in a conventional way.

Traditional demand theory is not really a theory of *individual* preferences but of *household* preferences. While not analyzing the nature of the amalgamation of individual preferences into a group preference, it is the latter that becomes reflected in market demand. We need not attempt a systematic analysis of the problem here*, but most formulations can be built upon by describing «household preferences» as being expressed on behalf of the members of the household. A child, being a member of his parents' household, has his preferences expressed *for him* by others, whatever the actual influence he has in the making of those household preferences. Moreover, the household's expression of preferences *on his behalf* is a normatively *legitimate* representation. So there is no problem of interpersonal comparability. We simply *treat* the preferences of the various individuals which make up each household as a unitary set.

* See J. Rothenberg, *The Measurement of Social Welfare*, Chapter 10, for a treatment of this issue.

But intertemporal resources allocation decisions which a parent makes *on behalf of* his heir(s) have this same feature. They *represent* the heir's interests. He is in effect still part of a household with respect to intertemporal decisions, even though not with respect to current decisions. This representation would seem to possess much the same normative persuasiveness as the use of household preferences to represent individual preference in present use.

There are differences between the Parent-Heir relationship and the Parent-Child relationship. Under the latter, for example, one could argue, at least to a good first approximation, that the Parent knows most of the alternatives of choice open to his child. In the former his knowledge about alternatives is conspicuously limited. He cannot predict technical change nor the constellation of allocation decisions by others. So he has only very imperfect knowledge about which resource transformations taken in the present will best enhance his heir's future situation. He is faced, therefore, with decision-making under uncertainty. But it is *his* uncertainty, for a decision for which *he* is presently responsible. No interpersonal comparability problem is involved. His decisions here have the same welfare significance as those of any individual planning for his own uncertain future. Indeed, given the crucial uncertainty about the length of one's life, the demarcation between planning for one's own and for one's heir's future is not at all clear-cut.

If knowledge about alternatives does not strain the analogy, the question of preferences is more troublesome. If a child disagrees with his father's prescriptions for him, we can at least say that «Father knows best» — or at least as a general rule does, or at least is supposed to. But even such a sometimes embarrassing crutch fails where father and son are both mature, and father's provisions for his son's future are at great variance with his son's own preferences. Some specializing of the Parent-Heir approach would seem to be able to approximate the persuasiveness of the Parent-Child relationship in this regard.

The context within which this is a problem is that, although the Parent is present at T_0 but not T_1, an aspect of him *is* present at T_1 and we are seeking to relate that aspect to his Heir in a way that will continue to reflect the Parent's preferences. We concentrate on the aspect of the Parent which remains at T_1, in other words, on intertemporal, intergenerational transfers (of wealth, income, commodities). There are three types of intergenerational transfers to consider.

First, transfers from parent to child when the latter is a minor. These generally do affect the child's subsequent development — remain as traces of the parent, therefore, even after he dies. But these are governed by the unity of purpose inherent in the familial «on behalf of» relationship. Second, transfers from parent to child as a bequest on the death of, or in contemplation of death by, the former. The issues are the size and composition of the estate. We assume that a single set of preferences — the parent's — allocates the parent's lifetime income between his own expected consumption levels and the expected size of the addition to his heirs' consumption levels through a bequest. This decision is made «on behalf of» his heirs, not in the sense that parent and heir would have chosen the same allocation, but in that the *legitimate* responsibility for allocating the resources of the original «family» between the interests of parent's and childrens' lifetime consumption levels is the parent's. Thus, it is the parent's concern for the children's welfare after his own death rather than a similarity of preferences about how that concern shall be expressed, that constitutes the normative linkage between the parent's well-being in T_0 and how his bequeathed estate is faring at T_1.

We further simplify the problem by assuming that the composition of the estate to be bequeathed is chosen to maximize its capital value, subject to the levels chosen by the parent for his own lifetime consumption. In this, both parent and prospective heir share preferences.

The third type of intergenerational transfer involves a gift from parent to child when both are mature, but not in contemplation of, or on occasion of, death. Here is a maximum opportunity for divergence of preference, but we assume this type of transaction is the least important from the point of view of population changes. Since both parties to the transfer are legitimate decision-makers in their own right and remain in the population, group welfare judgements are straightforward. The transaction is not «intertemporal» in our special sense. So preference divergences do no damage to the analysis.

In effect, then, our welfare comparisons between T_0 and T_1 involve linking the Parent's real wealth change (in terms of his utility function) between T_0 and the point of contemplation of death (which we may simplify by assuming that it corresponds to the «moment» before death), with the change in the capital value of the bequeathed estate (in market terms) from the «moment» of death to T_1. In other words, we pretend that the interests of the Parent in his legacies after death are still attended to by his Heir as a form of symmetry to his Parent's earlier concern for his interests. This conception of a representation of interests after the Parent has left the population as a physical decision-maker is not too unlike the more traditional conception of a representation of the child's interests before he has entered the population as a formally legitimate decision-maker.

Let us now examine the implication of this approach for the total impact of present resource decision on an Heir A′ with respect to his future situation, in order to discover what issues of interpersonal comparability remain from «intergenerational resource shifts». Time T_0 is the present, where both a and a′ are alive; T_1 is the future where A′ but not A is alive. At T_0, there are three kinds of decisions taken which will affect A′ at T_1: (1) decisions by A′, affecting himself; (2) by A, affecting A′: (3) by third parties, affecting A′. The first type involves no problem of intergenerational transfer or comparability. The second involves an intergenerational transfer, but, if we employ the method suggested in this section, that A's actions are taken *on behalf of* A′, then there is no problem of comparability. A's preferences and those of A′ are linked.

The third is more complicated. First, what is included in this category are both pecuniary and real external effects of third party actions. Third party behaviour affects the absolute and relative future availability of commodities, as well as such availability for inputs complementary and substitutive with those to be possessed by A′ at

T_1. In other words, third party behaviour at T_0 helps determine the economic significance of A′ ’s initial endowment at T_1: the set of employment and consumption alternatives available to him. Insofar as «third parties» include members of A’s generation, or other generations earlier than that of A′, intergenerational transfers and comparability will be involved.

The problem differs for pecuniary and real external effects. Pecuniary effects on A′ of some third party’s (B) actions at T_0 show up in market signals at T_0. They can therefore be adjusted to on the market, along with the rest of the market environment, by both A′ and A in making their plans for the future. Indeed, these influences cannot be isolated from the whole welter of market forces to which A and A′ are constantly adjusting in their market transactions. But real externalities stemming from B’s present actions do not show up in market signals. Market transactions cannot balance the relevant real marginal costs and benefits: the adjustment by A and A′ is incomplete at best. Thus, intergenerational transfers are involved, and the issue of comparability is raised, because neither A′ nor the guardian of some of his intertemporal interests, A, can fully adjust to these spillovers between T_0 and T_1.

In sum, no important problem of intergenerational comparability arises under the treatment so far sketched out except where real intertemporal externalities are involved. Ordinary market decisions with intertemporal impact can be treated as having the same welfare significance for the «future» generation as for the «present» generation. That two different populations are involved does no damage to the conventional welfare analysis. Only real externalities appear to make a difference.

Traditional treatments of the intertemporal resource allocation problem employ the terminology of time discounts and marginal rates of time preference. In these treatments it is frequently argued that the core of the problem of intergenerational comparability is that government has a deeper responsibility for the interests of future generations than does the private sector. This differential responsibility has as its reflection that the social rate of time discount is lower — significantly lower — than the private rate of time discount. It is instructive to examine some of the implications of our present treatment by seeing what light it throws on the question of time discount rates.

II

INDIVIDUAL AND SOCIAL TIME PREFERENCE

We have argued that insofar as real intertemporal externalities exist, private arrangements will be inadequate. If such externalities are substantial a collective mode of decision-making will be sought to internalize the externalities. But public decision-making will attempt to do what private decision-making would have been able to do in the absence of externalities. That is, it will represent an alternative mode by which *present* decision-makers act *on behalf of* the future generations. Government will in effect play the rôle of «secondary Parent» to the present set of Heirs.* It is useful, therefore, to compare the determinants of time preference for the primary Parents with those for the «secondary Parent» function of government.

Time preference is not a reflection of opportunity costs but of intertemporal preferences. It is a tool for making the benefits and costs experienced in different years comparable: therefore, for mak-

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* Remember that future heirs are to be related to the present set of parents through the linkage of the present heirs with their heirs, and so on down until «adjacent» populations are reached.

ing different time sequences of net benefits comparable — for sometimes enabling temporal sequences to be given a «present value». In order to isolate the time preference factor in private choices, we must abstract from differences in commodity combinations over time. If an individual is expressing his preferences among commodity combinations of this year's and next year's consumption (or indeed, for any two years' consumption), his choices will be partly determined by the specific composition of commodities within each bundle: there are intertemporal relations of substitutability and complementarity between commodities with different date, just as there are between those with the same date. Similarly, there may be non-neutral relations of this sort relating to the overall size of each dated bundle. Time preference is best seen, therefore, in the marginal rate of substitution at points representing equal overall consumption. This can be easily seen on a Fisher-type indifference map.

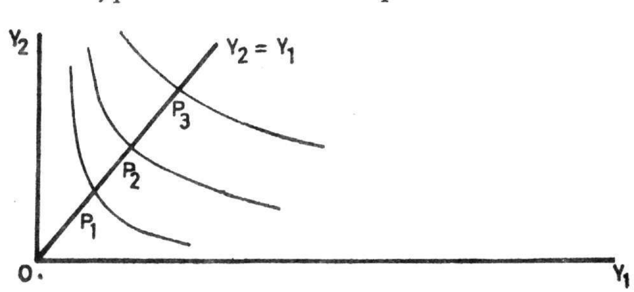

Figure I

Y_1 and Y_2 represent income of periods 1 and 2 respectively. I_1, I_2 and I_3 represent samples from three *alternative* preference systems (they do not belong to a single set of preferences). Line $OP_1P_2P_3$ is the ray showing equal two-year consumption.

At P_2 the marginal rate of substitution along I_2 equals 1. This shows zero time preference: marginal additions to period 1 income are equally important as marginal additions to period 2 income. At P_1, MRS>1. This shows positive time preference: period 1 income increments are worth more than period 2 increments. At P_3, MRS<1: negative time preference: marginal period 2 increments are more important than marginal period 1 increments.

In conventional analysis we assume that an individual (household) typically has positive time preference. This is attributed to a combination of the following:

(1) growing consumption needs over a life cycle;
(2) the possibility of death;
(3) impatience.

To what extent will these factors be present under government decision-making? Are there differences that would systematically lead to a discrepancy between public and private time preference?

The first factor can be largely disregarded even on the private level. While it may have real importance for each household at its own particular stage in the life cycle, the aggregate of households will involve households at all different stages of life and thus much cancelling-out will occur. The aggregate effect of these several vantages need not cancel to zero, of course, since it will depend on the joint distribution of income and life cycle stage — in particular, the rate and structure of population change. But this effect is so specific to assumed details that it is useful, and does not really affect the main argument, to assume that the aggregate impact on private intertemporal decisions is zero. The public decision is no different.

The possibility of death is a more complicated factor. For the individual, what is involved is the necessity to compare expected utilities. Assume for the moment that actual consumption next year has exactly the same utility significance for the individual as actual consumption this year, and that the individual can give up one unit of the latter at the margin *to make resources available for* one extra unit of consumption next year. But he is not certain that he will be alive to avail himself of next year's consumption option. There is a non-zero probability that, being dead, he will be unable to consume the increment next year. Let $U_i^k = U^k(Y_{ij}^k)$ be individual k's utility from consuming a certain income increment of size Y_i in period j (assuming a particular value of Y to which this is an increment). The function itself is unaffected by dating. Let Π = probability of living at least through next year. (We assume that a «year» is a single point of time to avoid the possibility of the individual dying during the «year». In this model, individuals die only between «years»). Then:

$$(1) \qquad U^k(Y^k_{11}) > \pi U^k(Y^k_{12}) + (1-\pi) U^k(0).$$

The expected utility to k from consuming increment Y_1 this year with certainty, exceeds the expected utility of a less than certain consumption of the Y_1 increment next year (where the income level to which this is an increment is the same as this year), the alternative being zero extra consumption for k. The marginal rate of substitution between *provision for* present and future consumption will be greater than 1: positive time preference.

This formulation is actually incomplete for our purposes. Individual k's next year's alternative to consuming increment Y_1 in case of death is not necessarily a zero utility situation: it is the provision of the extra Y_1 to his Heir k^1. Thus we amend (1) to the following:

$$(2) \quad U^k(Y^k_{11}) \gtreqless \pi U^k(Y^k_{12}) + (1-\pi)\, U^k(Y^{k^1}_{12})$$

Individual k will have positive, negative or zero time preference, insofar as the utility significance *to him* of his Heir's extra consumption is less than, more than, or equal to, that of his own comparable extra consumption.

Thus, the individual's time preference depends on interpersonal comparisons of utility performed by the individual himself. If we assume, as is generally done, that the individual's mortality makes present consumption more prized than the prospect of future consumption, this implies that we are tacitly assuming that the individual rates his own consumption as more important than comparable consumption by his Heirs. We shall make this assumption about the typical individual — in other words, about the aggregate of individual preferences.

It has often been argued that, while it is proper for individuals to be influenced by their own mortality, society as a whole need not be. The specific individual dies, but the population is forever replenished and continues indefinitely. The individual is mortal, but the society is immortal. As a result, the impact which mortality gives to a positive time preference for the individual is absent for the society. The private discount rate, being a reflection of individual decisions, each of which is informed by this concern for mortality, will be greater than the social discount rate, which need not make any adjustment for the mortality of the present population (it will be zero on this ground). It is instructive to examine this argument in terms of the approach we have so far outlined.

We assume first that government does not *per se* have more concern with future generations than do individuals presently alive. The present population delegates responsibility to government for certain intertemporal decisions because of an inability to make them efficiently on a private basis (e.g., because of the existence of important externalities). If a differential concern with the future emerges from this delegation, it must be because of the different structure of the decision-making situation, not because of differences in basic preferences between the population and «its» government. In short, we assume a fully representative government.

Analogous to the case of the individual, the government will have intertemporal preferences about *per capita* income bundles. Since actual allocations will involve absolute amounts of resources, those *per capita* evaluations must be accompanied by predictions of the future population. This is where individual mortality and its offset in birth — the dynamics of population change — enter. For the analogue to the individual evaluation, the government also compares two mathematical expectations. A possible shift in resources from present consumption will be judged in terms of the well-being of the present population, calculated under certainty, and the expected welfare of the prospective future population. An example of such a comparison is as follows:

$$(3) \qquad W\left\{\bar{Y}\left[N_{T_1}\right]\right\} \lesseqgtr \sum_i \theta_i \left[(S,D,R)_{i2} \Big|_{(N_{T_1})} \right] W\left\{\bar{Y}\left[(S,D,R)_{i2} \Big|_{(N_{T_1})} \right]\right\}$$

where: W is a social welfare function, expressing the social valuation of each distribution of real income to a population;

$(S,D,R)_i$ represents a particular successor population to the set (N_{T_1}) — the population at T_1:

S is the set of persons surviving from T_1 to T_2 — the Survival (or Overlap) Set;

D is the set of persons in N_{T_1} who died between T_1 and T_2 — the Death Set;

R is the set of persons added to the population between T_1 and T_2 by birth, with names of forbears from S and D (if the welfare function W distinguishes its treatment among R in terms of parentage; otherwise R can be treated as a set without parentage specified) — the Reproduction Set;

θ_i is the probability of successor population $(S, D, R)_i$;

$\bar{Y}[N_{T_1}]$ is the per capita income distribution of income enumerated by (N_{T_1}): i.e., $(\bar{Y}_1^1, \bar{Y}_1^2, \bar{Y}_1^3, \ldots, \bar{Y}_1^n)$;

$\bar{Y}[(S, D, R)_{i1}]$ is the per capita income distribution of the population $(S, D, R)_{i1}$.

The social walfare function here is simplified in the unusual way of making the *identity* of the recipients of income relevant to overall valuation but not the relative distribution of income among them, where the identity dimension is simplified to membership in S, D or R.

To isolate the key implication of this expected value comparison, we assume for convenience that the expected population size remains unchanged between T_1 and T_2. Then the mathematical expectation of new additions through birth equals the mathematical expectation of the number leaving the population through death. Since relative distributions within each group do not affect W, but membership in the different subsets of the successor population $F_2(N_{T_1})$ does, as soon as we specify a particular $\bar{Y}$ we specify a total income Y which is to be considered for distribution among alternate partitions of N_{T_1}; and the influence of any $\bar{Y}(S, D, R)_i$ on W may be equivalently rendered as $[\bar{Y} \cdot N(S),\ \bar{Y} \cdot N(D),\ \bar{Y} \cdot N(R)]_i$ where N(S), N(D) and N(R) are numbers of individuals within the respective subsets — i.e., as the set of total incomes of each subset: $[Y_S, Y_D, Y_R]$. Then we may express the total differential of expected welfare as:

$$(4) \qquad dE(W) = -\frac{\partial E(W)}{\partial E[\bar{Y} \cdot N(D)]} \, dE[\bar{Y} \cdot N(D)] + \frac{\partial E(W)}{\partial E[\bar{Y} \cdot N(R)]} \, dE\,[\bar{Y} \cdot N(R)]$$

with $dE\,[\bar{Y} \cdot N(D)] = dE\,[\bar{Y} \cdot N(R)]$ by our assumed expected constancy of income and population. A term with subset S is omitted since income is shifted only between D and R: if an extra death is expected to occur, an extra birth is expected to offset it, by assumption, and vice versa. No marginal shift can occur between S and either D or R.

Thus, the expected social welfare shift depends only on the relative welfare significance of members of the present (T_1) vs. the future (T_2) generations. It depends on an interpersonal comparison of social utility. Suppose (4) is negative in a particular instance. Then this means that the present generation is ranked higher than the future. Hence (3) would show:

$$(5)\quad W\left\{\bar{Y}[N_{T_1}]\right\} > \sum_i \theta_i \left[(S,D,R)_{i2}\Big|_{(N_{T_1})} \right] W\left\{\bar{Y}\left[(S,D,R)_{i2}\Big|_{(N_{T_1})} \right]\right\}$$

This would show positive social time preference. In other words, the existence of positive or zero social time preference depends on the same kind of interpersonal comparison between two generations as was the case for the individual decision-maker.

In a perfectly representative government, what determines this interpersonal comparison? This kind of government represents those currently living: the present constituents. If every such constituent individually ranks himself more important than his heirs (positive individual time preference), are there grounds on which he will transmit a different relative valuation for purposes of government action? We have argued that the main reason voters delegate responsibility for intertemporal action to goverment in this kind of (artificial, to be sure) system, is to accomplish what *they* would like to but cannot on a private basis. And this frustration of private behaviour relates to externalities or market imperfections, not to interpersonal evaluations. So it would seem that the voters would try to transmit the *same* relative valuation to government despite the call for collective action. In this model, therefore, mortality has the *same* effect in time preference for individual and collective decision-making. The sheer physical persistence of government beyond individual mortality does not necessarily lead to a different treatment of intertemporal allocation by government than by individuals.

The third factor determining individual time preference is impatience. Here there may be a genuine difference between private and collective decision-making, even when the collective decision-making is informed by the same values that shape individual decisions. The key here is the meaning of «impatience». The *same* experience is apparently preferred sooner rather than later. Why? We are now abstracting from the risk of dying. The individual *will* be present to partake of the experience. But he may not be the same individual later as he is today. There is a risk that he may have changed in such a way that the experience no longer is as satisfying as it would be, say, today, considering the sort of person he is today.

This is a slender reed on which to lean. There is as much chance that he will change so as to appreciate the experience *more* in the future as less. There is no good reason to believe that the probability distribution of personality change is anything but symmetric in this regard.

We might try a variant of the mortality issue. When the individual changes, whether becoming more or less appreciative, he becomes a different evaluative agent — a different person, in effect. As a decision-maker he becomes his own «heir». The present self has the same type of diminished moral responsibility for his future self as he does for a truly physically separate heir, although diminished in less degree. The individual is not quite as responsible for the person he might become as for the person he now is. Thus, time preferences here would be akin to mortality: he values his own — present — satisfactions more than those of his temporally-but-not-physically separate «heir».

Whether or not this type of attitude would qualify as «rational», in the sense used in utility theory, cannot be conclusively determined. Our early assumption about individual taste changes of course settles the question, but simply by begging it. Without reopening that discussion we may nevertheless admit, for purposes of the present discussion, that individual taste changes can occur. But, surely, there is a biological distinction which promotes a difference in degree large enough to be considered a difference in kind. While the individual sometimes changes in ways he did not anticipate and may even believe he could not fully control, surely his control over his own changes far exceeds his control over the changes of others. He *is* far closer to — and more responsible to — himself than to others; indeed, perhaps in an important sense just *because* he can change. It is because he *can* change that he must pay attention to the long-run consequences of his present actions upon himself. This is an epitome of responsibility.

So we reject the «heir» analogy for impatience. We are left with impatience as — impatience. The individual just cannot wait to have his fulfilling experiences. He finds himself giving up more for less just so that he can have it sooner. But eventually, by our assumption, the future does come for him and he finds himself with the lesser. Just as the future looks smaller from the vantage of today, so the past looks smaller from the vantage of today. He now regrets that he depleted the now-present future for the now-past present. This is most important. Impatience leads to inconsistent preferences between two time periods, since its valuations depend uniquely upon the temporal vantage, and that vantage necessarily differs between the two periods, all else concerning his tastes remaining unchanged. The systematic regret over past choices is the sign of inconsistency — irrational choice.*

Impatience then, may well be a situation in which the individual, when he acts upon his short-run preferences, acts inadvertently with respect to time in terms of his long-run preferences. If he could, he would *like* to be able to postpone satisfactions, but he cannot. In such a situation he may be able to act more in accordance with his own long-run utility function by delegating power to a third party either to act for him or to induce (or even compel) more appropriate short-run behaviour from him. Government may be that third party; and its power to commit resources with important intertemporal implications — or simply to affect market signals that strongly influence private decisions — may represent just this kind of implicit delegation. Then the public *wants* collective decisions to be based on different evaluations than those on which they base their daily private decisions.

If this is so, we should expect collective decisions to be systematically less impatient (if at all) than private decisions, to exhibit less positive time preference than the latter, for every hypothetical

* I have argued elsewhere that systematic regret may be the only dependable operational sign of irrationality — the multidimensionality and conceivable temporal changes in satisfactions making it otherwise possible for an outsider to attribute to almost every human choice an *ex post* «rationalization».

level of aggregate investment. The overall social time preference will not be zero insofar as mortality induces positive time preference in individual decisions, since private and social evaluations are similarly affected on that ground, as we argued above. Thus, social time preference should be treated as smaller than private time preference, the extent depending on the relative influence of mortality and impatience on individual time preference.

III

PUBLIC v. PRIVATE EVALUATION OF INTERTEMPORAL RESOURCE ALLOCATIONS

A. Impact of the Discrepancy Between Public and Private Discount Rates.

We now examine some implications of this discrepancy between social and private discount rates. At any given time, every decision-maker faced with alternative investment programmes (each one involving an intertemporal stream of payoffs and costs) should make them comparable by subjecting them to the same discount rate. As we noted earlier, the discount rate in this use is not a measure of opportunity costs (since these are simply the net payoffs to the remaining alternatives of any one programme), but simply a device for making payoffs and costs in different years comparable for the same programme, thereby collapsing multidimensional entities into a single dimension, and thus facilitating comparison across different programmes.

It may be objected that private decision-makers often use market rates of interest, not personal marginal rates of time preference, to discount future streams, and in this use they certainly do function as opportunity costs. In highly competitive private capital markets this may be appropriate, because in such markets, marginal productivity and time preference equal the market rate as an equilibrium condition; and where this is so it is not inconsistent with the procedure we suggest. It is simply a different formulation, since the market rate is ultimately compared with the subjective rate before a resource decision is taken. In these circumstances it does represent, however, a short-cut procedure. Such a short-cut is not appropriate, however, with governmental intertemporal decisions. For the rate at which the government can borrow is not a good measure of the social opportunity cost of the resources involved. It is too bound up with purely financial influences. The private resource investor does not have to ask himself whether government would make better use of the resources than he before deciding where to use them; but government does have to ask this before deciding whether any government use is justified at all. So the problem oi explicit comparability between public and private alternatives must be faced. The method we suggest is designed for this purpose.

The discrepancy between public and private discount rates does *not* mean that the government will subject its own prospective investment programmes to the social discount rate while comparing them to private alternatives subjected to the higher private discount rate. Such a procedure would erroneously inflate the productivity of resources in public relative to private investment. What the discrepancy does mean is that private investors will make resource decisions using the private discount rate for all the alternatives available to them, and government will make resource decisions using the lower public rate for *all* the alternatives available, either to public or private use. Thus the discrepancy does not greatly affect the overall level of resources going into the public sector instead of the private. It affects rather the relative evaluation among alternatives for the public and private decision-makers. For any pair of alternatives, for example, it may induce a different relative evaluation from public investors than from private investors. Projects with a higher proportion of their payoffs delayed to a remote future are more highly evaluated under the lower social discount rate than under the private rate. Less future-oriented projects are more highly rated under the private rate than under the public. Thus, for given overall allocation of investment resources between public and private sectors, public projects will generally be more future-oriented than private projects, all other things being equal.*[1]

An area on which this discrepancy has important impact is the field of «conservation». We shall examine the nature of the governmental responsibilities in this area from the point of view of the intertemporal allocation analysis presented here.

B. An Application: Government conservation activities.

The key feature of conservation is that it refers to resources (a form of capital and land) which are conspicuously depletable. Under actual or impending types and rates of utilisation, they can cease to exist in their present form in a near future. At the extreme, where there exists only one or a few units of the resource, depletability takes the form of uniqueness. There is only one Acropolis.

But impending depletion (even where uniqueness is involved) is not a sufficient condition for conservation to be relevant. Few people wish to conserve an obsolete piece of machinery, no matter

* *Ceteris paribus* is a necessary qualification, since the sector averages will be influenced by the average future-orientation of projects for which government responsibility is warranted. It might be, for example, that the most heavily future-oriented projects are far more efficiently operated on a private than on a public level.

1. It is possible that the government will employ its monetary policy to equalize the market rate with the social discount rate (for given aggregate investment schedule). If so, the impacts discussed here will not occur.

how unique. Further, the phasing out of some resource ordinarily raises no alarms, even where the resource retains its competitive serviceability, so long as there exist close substitutes for it. Conservation does not become an issue unless the resources whose depletion is threatened have unique serviceability. They are useful and, at least for some of their functions, have no close substitutes. But even then there may be no issue.

The serviceability which is unique must be thought to be especially important, in a way that is not reflected closely by market prices. This could either be due to attribution of some social valuation which differs from private valuation (about which we shall speak below), or, more simply, that its average product is considerably higher than its marginal product, upon which market price depends. For example, conservation often is an issue with water despite its low marginal valuations, since a substantial diminution of its availability for, say, drinking would drive its price quite high.

Most such unique depletable resources are privately owned. Decisions as to how the resources should be expended are private decisions, like other private decisions on resource utilisation. Every such decision commits the future by helping to determine an overall pattern of resource utilisation and thereby determining the set of productive possibilities open to future populations. More particularly, every private resource decision simultaneously establishes one resource use *and* precludes others. That the specific subset of such decisions which deals with unique depletable resources precludes future benefits from these same resources is therefore not at all unusual, since every resource decision is preclusive. Are there any special grounds then for government to step in to influence or actually take responsibility for decisions involving unique depletable resources?

A first ground is, of course, the existence of substantial externalities in the use of such resources. If the Acropolis and its structures are preserved from mineral exploitation or from subdividing for «view lots» housing development so that I may be able to enjoy it, then my neighbour finds it available to him to enjoy as well. Externalities are a general and well-known ground for government assumption of responsibility for resource decisions. We shall take it for granted in the present discussion. Even so, we must note that the externalities argument does not answer the really critical question in a case like that of the Acropolis. The benefits may be diffuse, but they are geographically limited enough so as to vest some property rights in them. Many prospective beneficiaries can be forced to establish a separate transaction with the property owners in order to enjoy benefits. A private enterprise, or even the government, may «own» the Acropolis and extract revenues from the sale of services. In particular, government ownership can internalize whatever externalities exist, and even finance the resource use by methods which do not depend upon *quid pro quo* control over supply.

The main question for our purposes is not the possibility of owning something which can produce a saleable commodity, but rather what commodity it should produce. Should evidence of mineral deposits be detected within the Acropolis hill the question would then arise, should the complex be maintained in its present state for historical and archaelogical observation and inspiration or converted into a congeries of mines and mine shafts? Should the structures, as irreplaceable and unique as the land itself, be removed from their present site to permit alternative use of the land for housing developments? These questions are actually of the same form as: should petroleum be produced at a rate which would exhaust presently known resources in 10 years or 20 years or 50 years? Thus, the question is not directly, who shall own or operate enterprises employing the resources in question but rather, what resource decision should be made? It turns out that the latter question *indirectly* affects the former.

It is sometimes argued that public resource decisions in the area of conservation are often based not on private values but social values. While the population may not really prefer inspiration and an archaelogical wonderland to the additional metal products (or housing products) as individual consumers, they do as members of the society as a whole — or they do as guardians of the future generations. The relevance of guardianship is likely to be misleading. The alternative of investment in steel producing facilities (or whatever) is not obviously less future-oriented than maintenance of this historic shrine, since the former can initiate an indefinite temporal sequence of increased investment in productive capacity. But the notion that there are social valuations of inspiration, of history, of scenery, and the like which differ from the corresponding individual valuations is the critical one, and it is controversial.

I believe that judgements of this sort do exist. In effect, they involve the individual voter saying to himself: I personally am not the least bit interested that this most perfect of buildings, this most beautiful vestige of our pagan past, this triumph of art and skill may all disappear; but I believe that these involve inspiring interests and I should like to live in a society in which outlets for interests such as these are available.

The crux of such social values as may exist in the conservation area seem to be, not to *force* others to some particular form of consumption (commodity x is bad for you: you must not consume it; commodity y is good for you: you must consume it), but rather to *keep open the option* of consuming a certain commodity. The value becomes operative when private decisions threaten to make this option disappear for the entire society in the near future. The value takes the implicit form: current circumstances seem to suggest that present utilization of a type or at a rate that endangers future availability is preferable to that future availability. But this judgement *may* be wrong. If so, it will be too late to reverse our commitments after we discover our error. Let us therefore err on the side of conservation, since the values that would be sacrificed are especially worth while and to some extent unique.

In this formulation conservation is a combination of a normative «social» approbation of particular resource uses and an attitude of conservatism toward planning for an uncertain future. The trouble is that not everyone will agree to attribute the special «social approbation» to the same resource uses. Most individuals have pet favourites, but they do not overlap much. Any particular candidate for such approbation is likely to be highly controversial. Thus, a government conservationist mandate will be basically controversial if based on this formulation. It is possible, however, to drop the controversial social valuation prop and by applying our earlier analysis of public and private discount rates, to derive much the same conservationist prescriptions for government. In other words, many conservationist activities of government can be rationalized without resorting to assertions that public decision-makers know more than private decision-makers, or that they are the guardians for future generations, or that certain resource uses are more valuable socially than they are privately.

The key characteristic of the resource uses we are speaking of is that at least one of the options is essentially irreversible. For example, while a current decision to maintain the Acropolis in its present status holds open the option for extensive mining (or real estate) operations in the future, a decision to engage in heavy drilling and blasting now (or dismantling the structures now) effectively forecloses the option for returning the site to its former status in the future. It is the fact that irreversibility occurs in the context of planning for an uncertain future that opens the way for a rationalization for conservation.

Planning the intertemporal use of resources is not a one-shot action. A decision is made today on how to commit certain resources on the basis of presently available information. But tomorrow additional decisions will be made on the basis of new information, some referring to the commitment of new resources, and some reviewing today's commitments on the basis of the new information. Today's decisions are neither definitive nor are they generally irrevocable. Thus, present decision makers must make their decisions with an eye toward the possibility that the future may not be as they predict today, and that where such discrepancies occur, modifications may be called for in resource utilization. The possibility of making such modifications is an additional source of payoff for each option. We shall show the impact of this by presenting a formal model.

1. Alternatives of choice

Suppose there are two alternative resource uses possible, as follows:

(1) A — use the resource in an irreversible manner. This is the non-conservation use. In our model we shall refer to it as the «shift» from conservation to non-conservation use.

(2) $\bar{A}$ — use the resource in a reversible manner, i.e., a manner which keeps open the possibility of making the shift to A at any time in the future. This is the conservation use. We shall refer to it as the «non-shift».

Thus, conservation consists in keeping open the possibility of changing the resource to its alternative use at any time in the future; non-conservation consists in foreclosing the possibility of changing the resource to the use it once had. The essence of the model rests in this asymmetrical «reversibility».

We use the following notation:
T_0 to mean time present; T_i to mean some future date (period i); t_i indicates the interval $T_i - T_0$; A_i ($\bar{A}_i$) is the shift (non-shift) at time T_i.

Only one decision is made in this model: the choice of A_0 vs. $\bar{A}_0$. This is made at T_0, the point of decision. Considerations about future circumstances throw light only on this decision, not on a decision at T_0 about some A_i vs. $\bar{A}_i$. This is a sequential decision-making model. The decision between A_i and $\bar{A}_i$ is made only when T_i itself becomes the present through the passage of time.

Decision makers:

α) private sector

β) government sector

γ) undifferentiated (general) decision-maker.

Discount rates of α, β: $d_\alpha > d_\beta$ (as a result of Section II).

Each decision maker (γ) selects an alternative on the basis on his objective function Φ — the present value of each option. Φ_A differs from $\Phi_{\bar{A}}$ because of the asymmetrical reversibility:

(1) a) $\Phi_A = P_{A0}$

b) $\Phi_{\bar{A}} = P_{\bar{A}0} + R$

where P_{A0} is the present value at T_0 of the net payoffs from A which are expected at T_0;

$P_{\bar{A}0}$ is the present value at T_0 of the net payoffs from $\bar{A}$ which are expected at T_0, if $\bar{A}$ is continued for the entire lifetime of the resource; R is the value at T_0 of the reversibility option: i. e., that $\bar{A}$ can be shifted to A in any subsequent period, but not vice versa (hence there is no R term for Φ_A).

Φ_A and $\Phi_{\bar{A}}$ can be thought of as the capital values of ownership in each type of project. If $\bar{A}$ is chosen one obtains along with it an option to change project type in the future. It is a joint purchase of a commodity bundle comprising a project type and a flexibility (an option can be taken up at any time).

(2) Assume $P_{A0} > P_{\bar{A}0}$.

This suggests that A_0 be chosen unless $R > P_{A0} - P_{\bar{A}0}$. The model examines the conditions under which $\bar{A}_0$ should be chosen even though $P_{A0} > P_{\bar{A}0}$ — in other words examines what determines the value of R.

2. The Expectation of New Information

What is the consequence for decision maker γ if he chooses $\bar{A}_0$? At T_0 γ knows that the passage of time will bring him new information bearing upon the relative profitability of $\bar{A}$ and A over their productive lifetime. At each T_i there will be a best set of estimates of P_A and $P_{\bar{A}}$, just as P_{A0} and $P_{\bar{A}0}$ are the best estimates at T_0. But γ doesn't know with certainty what these best estimates will be. He does, however, have a probability distribution of what these best estimates will be at any T_i. (The two probability distributions for T_0 collapse respectively to the points P_{A0} and $P_{\bar{A}0}$).

Since at T_0 his best estimates (point estimates) are P_{A0} and $P_{\bar{A}0}$, we assume he has no reason to *expect* that the best estimates of P_A and $P_{\bar{A}}$ will diverge *systematically* from P_{A0} and $P_{\bar{A}0}$. So the *means* of the probability distributions of best estimates of P_A and $P_{\bar{A}}$ are P_{A0} and $P_{\bar{A}0}$ respectively, and the distributions are symmetric around these means.

Define these probability distributions for T_i (as expected at T_0) as G_{Ai} $(P_{Ai};\ P_{A0},\ \sigma_{Ai})$ and

$$G_{\bar{A}i}\ (P\bar{A}i;\ P_{\bar{A}0},\ \sigma_{\bar{A}i}) \qquad i = 1, 2, \ldots$$

where PAi, $P\bar{A}i$ are the variates of the distributions, P_{A0}, $P_{\bar{A}0}$ the respective means, and σ_{Ai}, $\sigma_{\bar{A}i}$ the respective standard deviations. For simplification we assume that G_{Ai} and $G_{\bar{A}i}$ are independent distributions. This is not strictly true. However, interdependence does not damage the argument, it simply complicates the mathematics.

Now, we characterize γ's anticipations about new information by assuming that for periods close to the present the information is not likely to diverge much from information available today, but that as one contemplates more and more remote periods the information may be more and more at variance with today's. In other words, for the near future the G distributions are likely to cluster closely around P_{A0}, $P_{\bar{A}0}$—very small σ_{Ai} and $\sigma_{\bar{A}i}$; but as a more and more remote future is considered the bunching becomes weaker and weaker — σ_{Ai} and $\sigma_{\bar{A}i}$ become progressively larger. Thus,

$$\frac{\partial \sigma_i}{\partial t} > 0 \tag{3}$$

The extremest futures are associated with the extremest uncertainty: G_i approaches the uniform distribution with indefinitely large range.

3. The Value of the Reversibility Option at T_1

Two types of outcomes are made relevant by the prospect of new information at T_1: a) profitability turnabout, b) profitability of shift.

a) *Profitability Turnabout*

Suppose that *at* T_1 γ should discover that new information now resulted in new best estimates of lifetime returns from A and $\bar{A}$ such that

(4) $$P_{A1} \leqq P_{\bar{A}1}.$$

The new best estimates no longer favour A. This means that the capital value of ownership in A suffers a depreciation relative to that of $\bar{A}$. So the continuation of $\bar{A}$ no longer promises a disadvantage relative to A. The prospect at T_0 of such a turnabout occurring at T_1 with its relative capital appreciation for $\bar{A}$ should enhance the present value of $\bar{A}$ at T_0: in other words, it should help comprise a component of R. To compute the size of the benefit engendered to $\bar{A}$ at T_0 we must pursue some new relationships.

$$\begin{aligned} \text{Let } D_0 &\equiv P_{A0} - P_{\bar{A}0} \\ D_1 &\equiv P_{A1} - P_{\bar{A}1} \\ D_i &\equiv P_{Ai} - P_{\bar{A}i} \qquad \text{for all } T_i \end{aligned}$$

Now the condition for turnabout to occur is

(5) $$P\bar{A}_1 \geqq PA_1$$
$$\text{or } D_1 \leqq 0$$

and the improvement in $\bar{A}$'s capital value relative to A's is

(6) $$(P_{\bar{A}1} - P_{\bar{A}0}) - (P_{A1} - P_{A0}),$$

or, by rearranging terms:

(7) $$D_0 - D_1.$$

These relations are shown on the illustrative diagramme, figure 2.

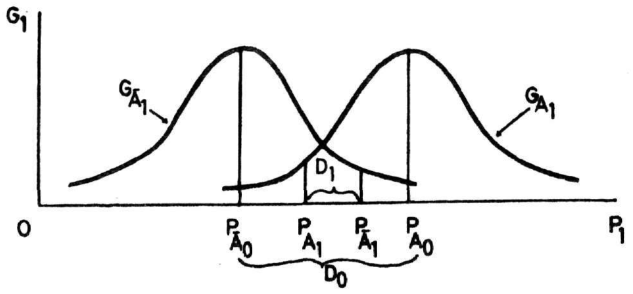

Figure 2

For simplicity we assume G_{A1} and $G_{\bar{A}1}$ to be normal with equal variances in Figure 2.

This particular turnabout is only one such possible turnabout. There are many pairs of $P_{\bar{A}1}$, P_{A1} which give $D_1 \leqq 0$ and for any of these $\bar{A}$ gains a relative capital appreciation. To find the $\bar{A}$-enhancing value of all of these possibilities we calculate the mathematical expectation of gain from turnabout. We do this by defining the joint probability distribution of G_{A1} and $G_{\bar{A}1}$ at T_1: $J_1(P_{A1}, P_{\bar{A}1})$. So the benefit to Φ_A at T_0 from a potential turnabout at T_1 is the expected value of that portion of $J_1(P_{A1}, P_{\bar{A}1})$ where $D_1 \leqq 0$, i.e., $J(P_{A1}, P_{\bar{A}1}; D_1 \leqq 0)$.

Let Π_1 be the density function of J_1.

Then the required expected value is:

$$(8)\quad E\{D_0 - D_1 / D_1 \leqq O)\} = \int_{-\infty}^{\infty} (D_0 - D_1)\ \Pi_1\ (P_{A1j},$$

$$P_{\bar{A}1j}/D_{1j} \leqq O)\ dP_{A1}\quad dP_{\bar{A}1} = E(D_0\text{-}D_1)_v\ \Pi_{1v} = v_1$$

where $E(D_0\text{-}D_1)_v \equiv$ mean turnabout gain
$\Pi_{1v} \equiv$ total probability of turnabout in J_1.

b) *Profitability of Shift*

Suppose that at T_1 γ should discover that new information still showed A with higher expected lifetime returns than $\bar{A}$: i.e., $D_1 > O$. So the choice of $\bar{A}_0$ still appears costly. But $\bar{A}$ is reversible, so γ might decide to shift from $\bar{A}$ to A at T_1, thereby offsetting the loss of having selected $\bar{A}$ from T_0 to T_1. Such a shift, by switching his assets to a higher earning stream, would enhance the capital value of γ's holdings, which was P_{A0} at T_0. This too, as under a turnabout, results in capital appreciation for γ at T_1.

For every $D_1 > 0$, γ stands to gain by switching to A. But the size of this gain depends on a number of factors. First, however large D_1 is, the gain from switching at T_1 can *at most* equal the original expected loss at T_0 from selecting $\bar{A}$, since γ is not better off from having been operating $\bar{A}$ rather than A between T_0 and T_1. At most he may be no worse off by changing than if he had chosen A at T_0.

Second, the benefit from switching is a function of the size of D_1 and the duration of time γ has operated $\bar{A}$ instead of A. It is a positive function of the first and a negative function of the second.* The larger D_1 is, the greater is the differential in returns and therefore the gain is greater from switching to A. As to the effect of t, γ has operated the inferior project $\bar{A}$ instead of A from T_0 to T_1. Since we assume that all projects have finite lifetimes**, he has lost profitability opportunities which are irretrievable. He cannot wholly offset his already-sustained differential losses. The larger t is the larger is the part of the productive lifetimes of both projects which is past and the smaller is the part in which γ can hope to offset his earlier losses by shifting.

Third, the act of switching itself is not costless. There is a real resource cost of converting the re-

* It might be a *positive* function of t if $\bar{A}$ and A were related as input to output, so that operation of resource use $\bar{A}$ *enhances* the productivity of the resource in subsequent use as A. An example is if the continued pattern of pilfering, depradation, and erosion of the Acropolis when allowed to remain in its aesthetic-historical-archaeological use increased, making it easier to clear it for real estate development or mine it for mineral deposits for which its first use was to be destroyed. But we are assuming that the alternative uses are exclusive, not complementary. So this phenomenon will not occur in our model.

** This is not always necessary for our purposes, since the need to discount future returns to a present value status means that even infinite streams will have a finite economic horizon insofar as successive payoffs do not increase as fast as the discount factor.

sources from one usage to the other. Some conversions are very costly, some involve only trivial using-up of resources, depending on details. The present version of our model does not depend on any special features of the conversion cost function, so we shall simplify the exposition and assume that C is constant over time.

We may now summarize the payoffs arising from a shift out of A into $\bar{A}$ at T_1. For each $D_{1j}>0$, the benefit from such a shift is:

(9) $b_{1j} \equiv B(D_{1j}, t_1, t_F / D_{1j} > 0)\ (D_0/D_{1j}) - C(t_1)$

where b_{1j} is the net benefit;

$B(D_{1j}, t, t_F)$ is the gross benefit function, with $t_F =$ total productive lifetime of A and $\bar{A}$ (assuming them equal), such that

$$\partial B / \partial D_{1j} > 0,\ \partial B / \partial\ (t_1/t_F) < 0 \text{ and}$$
$$B(D_{1j} \mid t_1/t_F > 0) < D_{1j};$$

$C(t_1)$ is the conversion cost function, temporarily assumed a constant.

D_0/D_{1j} is the normalization factor to place all gains from the shift within the context of offsetting the original loss D_0 (thus, if $B(D_{1j}) = D_{1j}$ and $C(t_1) = 0$ — the case of complete offset — then $b_{1j} = D_{1j}\ (D_0/D_{1j}) — 0 = D_0$). $\partial b_{1j} / \partial D_{1j} > 0$ (we assume that $\partial B/\partial D_{1j} > 0$ and $\partial^2 B/\partial D_{1j}^2 > O$; i.e., a rising D_{1j} causes B to rise at an increasing rate).

But:

(10) $b_{1j} > 0$ if and only if $B_{1j}\ (D_0/D_{1j}) > C(t_1)$.

Thus, even with positive B_{1j}, the shift will not be worth anything unless its normalized gross value exceeds the marginal cost of making the shift (since $C(t_1)$ is incurred if and only if a shift is actually made). So there can be a gain from shifting only where D_{1j} is high enough to fulfill (10).

Each occurrence of a D_{1j} high enough to fulfill (10) makes a shift profitable. If the shift is made, then, as under profitability turnabout, γ experiences a capital gain in his ownership over the initial value $P_{\bar{A}0}$. The overall worth to γ of the possibility of a shift at T_1 (from the vantage of T_0, not T_1) is the expected value of all such possible capital gains. This is the mean gain from that part of the joint probability distribution J_1 where each D_{1j} is large enough to fulfill (10), times the total probability in that part of J_1: i.e.,

$$(11) \qquad E\{b_1 \mid b_1 > 0\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (b_{ij} \mid b_{ij} > 0)\ \Pi\,[P_{A1}, P_{\bar{A}1} \mid b_{1j}\,(D_{1j} \mid D_0,\ C(T_1)) > 0]\ dP_{A1},\ dP_{\bar{A}1} = E(b_{1s})\,\Pi_{1s} = s_1$$

where $E(b_{1s}) \equiv$ mean shift gain

$\Pi_{1s} \equiv$ total probability of outcomes where shift is profitable

(11) represents the expected (mean) appreciation in capital value at T_1 from the opportunity to

switch at T_1 if the choice of $\bar{A}$ still seems erroneous.

C. *Total Value of Reversibility Option at T_1*

Let R_1 be the value of the reversibility option with respect to opportunities anticipated for T_1. Then, from (8) and (11) we obtain:

$$R_1 = \frac{1}{(1+d)t_1} [v_1 + s_1] \tag{12}$$

The first term discounts the capital appreciations anticipated at T_1 back to T_0 to obtain a present value at T_0.

4. CALCULATION OF R

Calculate R_i for each T_i in the future. We obtain the sequence $\{R_1, R_2, \ldots\}$. Now, R is the maximal element of this sequence.

$$R = \max \{R_1, R_2, \ldots\} \tag{13}$$

R is the maximal element of the sequence and not some weighted average of it because of the fact that each element R_i refers back only to observations at T_0, not to actual accumulated observations between T_0 and T_i. The latter would be necessary, amplified by a theory of optimal non-sequential decision-making, if more than one element of the sequence were to appear in R. This is due to the fact that the flexibility option ends when an actual shift occurs, and to determine at T_0 when this will happen requires γ to choose *at T_0* a strategy of a sequence of choices between A and $\bar{A}$ on the basis of a sequence of actual new information. The present model requires that, on the basis of the information available to him at T_0, γ make a choice only at T_0. Later choices must wait upon the receipt of actual additional information.

Thus, the quantities R_i $(i = 1, 2, \ldots)$ do not reflect a true temporal sequence of happenings — from T_0 to T_1, then T_1 to T_2, T_2 to T_3, etc. — where it is particular linked subsets of these temporal intervals which represent the possible alternative gains. It is each entry R_i itself which represents the alternative characterizations of the capital gain possibilities inherent in the expectations about future information — the family of J_i distributions $(i = 1, 2, \ldots)$. If the asset owner is selling his prospect for capital gains under each set of such circumstances, it is the maximal element of $\{R_1, R_2 \ldots\}$ which will determine the price.

It is instructive to consider whether the sequence $\{R_1, R_2, \ldots\}$ will generally have a maximal element. Consider each term of R_i : v_i and s_i.

$$(14) \quad \frac{\partial v_i}{\partial t} = \frac{\partial [E(D_0 - D_i)_v \Pi_{iv}]}{\partial t} > 0 \quad \text{since} \frac{\partial \sigma_i}{\partial t} > 0.$$

With advancing t the dispersion of G_{A1} and $G_{\bar{A}1}$

and therefore of J_i increase; so the probability in both tails of J_i increases, thereby increasing the mean of each part of J_i for which the relevant capital gain occurs. However, Π_{iv} and Π_{is} sum to less than one and each to less than ½.

$$(15) \qquad \frac{\partial s_i}{\partial t} =$$

$$\frac{\partial[E\{B(D_{1j},t_1,t_F) \mid D_{1j} > 0)\ (D_0/D_{1j})\text{-}C(t_i)\}\Pi_{is}]}{\partial t} \gtreqless 0.$$

Less can be said definitively about the net effect of this term. The increasing dispersion of J_i here too acts to increase s_i. But there are offsets to this effect. Increasing dispersion operates through an increase in mean D_i (in the upper tail), but this has an impact on s_i which has a strict upper limit D_0 (as guaranteed by the term D_0/D_1; but we do assume $\partial s_i/\partial E(D_i) > 0$). Even more important, to make the problem interesting, we assume that the project has a fixed lifetime t_F.* Then B is a decreasing function of t_i and in many realistic cases a rapidly decreasing function. In addition, it is reasonable to suppose that $C(t_i)$ is an increasing function of t_i — in some cases, a rapidly decreasing function. Thus, in sum, s_i may or may not increase monotonically as t_i increases indefinitely. There are forces present which strongly suggest

* Otherwise one need never shift, except for the influence of the discount rate.

that a steady increase — if it occurs at all — will be quite moderate.

Thus, $v_i + s_i$ may not even be an increasing sum with t_i; but if it is, it is not likely to be a rapidly increasing sum (only quite special circumstances would bring this about). This is enough to establish a stronger presumption. R_i results from discounting $v_i + s_i$ to present value terms. The discount factor is a rapidly increasing function of t_i. It is likely to overtake the more slowly rising — if at all — $v_i + s_i$ for even moderate values of t_i. So a maximal element very likely exists for the sequence $\{ R_1, R_2, \ldots \}$ except in highly special circumstances.

5. PRIVATE VS. GOVERNMENT CHOICE

Sections 1 - 4 above establish that the criterion of choice between A and $\bar{A}$ (at T_0) is:

$$\text{(16)} \qquad \text{If } R > P_{A0} - P_{\bar{A}0} \qquad \text{Choose } \bar{A}$$

$$\text{If } R < P_{A0} - P_{\bar{A}0} \qquad \text{Choose A.}$$

We consider how the systematic difference between public and private decision making affects this calculation. To this effect we assume that both kinds of decision makers have the same real estimates of future consequences, but that they differ only with respect to the discount rates they employ.

We use the following notation. Let α and β refer to private and public decision makers; and d_α, d_β are their respective discount rates. We assume

$$(17) \qquad d_\alpha > d_\beta.$$

Although α and β have similar expectations, these might involve quite different present values because of the different discount rates used. What is relevant here are not absolute values of P_A, $P_{\bar{A}}$ over time but only values of $(P_A - P_{\bar{A}})$. We make the simplifying assumption that

$$(18) \qquad (P_{A0} - P_{\bar{A}0})_\alpha = (P_{A0} - P_{\bar{A}0})_\beta.$$

Consider the following sequences:

$$(19) \qquad \{R_1^\alpha, R_2^\alpha, \quad ..., R_i^\alpha, ...\}$$
$$\{R_1^\beta, R_2^\beta, \quad ..., R_i^\beta, ...\}$$

Each term R_i^α is a function of $(P_{Ai} - P_{\bar{A}i})_\alpha$; similarly each term R_i^β is a function of $(P_{Ai} - P_{\bar{A}i})_\beta$; and these in turn are a function of d_α and d_β applied to common expectations. If all $(P_A - P_{\bar{A}})$ expectations were positive, $(PA_i - P\bar{A}_i)_\beta > (P_{Ai} - P_{\bar{A}i})_\alpha$, because common expected differences in outcome in the future relative to some period T_i are being discounted by d_α and d_β respectively with respect to T_i as «the present». But these expectations are positive and negative, so $d_\alpha \neq d_\beta$ does not automatically prevent (18) from holding nor, more generally, (20), as follows, from holding:

(20) $(P_{Ai} - P_{\bar{A}i})_\alpha \lesseqgtr (P_{Ai} - P_{\bar{A}i})_\beta$ for *any* $i = 1, 2, \ldots$

We now argue that, despite $d_\alpha \neq d_\beta$, the sequences underlying R^α and R^β, namely:

$$(21) \quad \begin{aligned} &\{(v_1 + s_1)_\alpha, (v_2 + s_2)_\alpha, \ldots, \quad (v_i + s_i)_\alpha, \ldots\} \\ &\{(v_1 + s_1)_\beta, (v_2 + s_2)_\beta, \ldots, \quad (v_i + s_i)_\beta, \ldots\} \end{aligned}$$

are not systematically different in magnitude. Each v_i refers to expectations about the profitability of $\bar{A}$ being improved relative to that of A—D_i in the relevant tail is negative. For each such, $d_\beta < d_\alpha$ makes the relevant $P_A - P_{\bar{A}}$ effect *less* for the public decision maker than for the private. On the other hand, each s_i refers to expectations where the profitability of A is greater than that of $\bar{A}$—D_i in the relevant tail is positive. For each of these, $d_\beta < d_\alpha$ makes the relevant $P_A - P_{\bar{A}}$ effect *greater* for the public than for the private decision maker. Moreover, there is nothing in the structure of the v and s terms to suggest that the positive differential will differ systematically from the negative differential. Thus, the differential effects on present values produced by the different discount rates will tend to cancel out. At this level of generality we may therefore be entitled to assume that the sequences in (21) are term-by-term approximately similar. This is a key assumption and it derives not from an arbitrary simplification but from the underlying structure of the respective terms.

If the sequences in (21) are similar, then the sequences in (19) must be systematically different, for each term in (21) is discounted by the respective d_α or d_β. In fact, $\{R_1^\beta, R_2^\beta, \ldots, R_i^\beta, \ldots\}$ dominates $\{R_1^\alpha, R_2^\alpha, \ldots, R_i^\alpha, \ldots\}$ since $d_\beta < d_\alpha$. So the maximal element of the first must exceed that of the second. Thus,

$$(22) \qquad R^\beta > R^\alpha.$$

In other words, the reversibility option is worth more to the public decision maker than to the private. Given the same two projects, with similar expectations about future consequences (and similar valuation of «outputs» and «costs»), the asymmetric opportunity to reverse the decision at some time in the future is more likely to make the public decision maker choose the conservation alternative than the private decision maker.

Thus, *there are circumstances* (it is not always so) where the private sector, when faced with a choice between a conservation alternative and a non-conservation alternative, would choose the latter, while the public sector, *using the same valuations and expectations*, would choose the former. The social judgement, due to the lower discount rate, is more conservation prone. This is *not* the same thing as saying that future contingencies — in brief, the Future — are more important for a decision maker with a lower than one with a higher discount rate. It is the asymmetric ability of the two alternatives to cope with future contingencies, and the fact that in this context the use of different discount rates does not systematically offset the value of this asymmetric opportunity, that is at the heart of the present demonstration.

The policy areas where such differences in future flexibility are most likely to occur are where the alternatives differ most markedly in the extent to which they preserve future options for reversals, modifications, etc. Conservation is an exceptionally auspicious example of such differences, as we noted earlier. Here are activities where private decision-makers, led by profitability considerations (with some attention to irreversibilities: small R's) sometimes seem to the government to be paying inadequate attention to the foreclosing of options for the future. The government is therefore led, either systematically to alter private incentives for resource utilization (through taxes, subsidies, regulations) or, where geographic concentration permits and unique resources are more imminently irreversibly imperiled, to assume outright ownership and operational responsibility.

A qualification of this result must be mentioned.

The argument rests on the relative attractiveness of flexibility for government and private decision-makers as a hedge against uncertainty. But the uncertainty against which they seek to hedge is not entirely out of their control. Some elements of uncertainty can be controlled or eliminated by means of large-scale operations, diversification and structural changes in the economy (for example, efficient monetary and fiscal policies). Government is in an especially good position to control or lessen uncertainty because, in addition to the possibility of operating on a very large scale, it can also, by new legislation, produce structural changes that decrease the variability of relevant outcomes. But these powers provide a substitute for flexibility as a response to uncertainty. Insofar as the government uses its power directly to affect the sources of uncertainty, it need make less provision for flexibility. Thus, it may not always choose flexible alternatives even in areas where it already has operational responsibility. And its response to private depleting decisions may not be direct conservation intervention but rather indirect substitutes. Thus, in the field of petroleum mining, in addition to tax subsidization, assertedly for conservation purposes, the Federal Government of the United States spends resources to develop alternative sources of power for the future (nuclear energy).

In sum, aspects of government conservation practices can be rationalized, along with explanations in terms of social values vs. private values, as responses to secure flexibility in the face of uncertainty and of a private propensity to accord less importance to flexibility than the government deems warranted because of a higher than social rate of discounting future benefits. Uncertainty does not always lead to government intervention in the market to secure a more flexible policy. Sometimes the government will eschew the conservational approach partly or totally, and directly attack the sources of uncertainty. Included under this rubric is the investment in new resources or technology to substitute for what is currently being depleted.

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