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## Distributed optimization in public economics via decentralized public finance: A proposal for the spatial distribution of public goods and services

Ioannis Papastaikoudis, Prodromos Prodromidis,
Jeremy Watson and Ioannis Lestas

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**Ioannis Papastaikoudis**
Visiting Scholar, Centre of Planning and Economic Research (KEPE), Athens, Greece
Postdoctoral Researcher, Judge Business School (CERF), University of Cambridge, Cambridge, United Kingdom. Email: ip352@cam.ac.uk

**Prodromos Prodromidis**
Senior Research Fellow
Centre of Planning and Economic Research (KEPE), Athens, Greece
E-mail: pjprodr@kepe.gr, Tel.: (+30) 210 3676412

**Jeremy Watson**
Senior Lecturer
Faculty of Engineering, University of Canterbury, Christchurch, New Zealand.

**Ioannis Lestas**
Professor
Department of Engineering, University of Cambridge, Cambridge, United Kingdom.

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**DISTRIBUTED OPTIMIZATION IN PUBLIC ECONOMICS VIA DECENTRALIZED PUBLIC FINANCE**

**A proposal for the spatial distribution of public goods and services** •


**Abstract**

The paper proposes a new approach in public economics, in a decentralized finance setting, by using distributed optimization techniques to help plan inter-regional and intra-regional public goods and services for multiple regions, each region with its own budget, natural and population characteristics. The goal is to provide policy makers with a planning optimization tool for public infrastructure spanning over a number of regions in a way that improves the welfare of its constituent populations. To that end it treats the problem as a utility maximization problem and calculates the Marshallian demand for public infrastructure.



**Keywords:** Network optimization; Decentralized public finance; Utility maximization and demand functions; Regional, subregional and local policy planning; Public infrastructure, goods and services



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# ΔΙΚΤΥΟΚΕΝΤΡΙΚΗ ΒΕΛΤΙΣΤΟΠΟΙΗΣΗ
# ΣΤΗΝ ΔΗΜΟΣΙΑ ΟΙΚΟΝΟΜΙΚΗ
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Μια πρόταση για την χωρική κατανομή των δημόσιων αγαθών και υπηρεσιών

Ιωάννης Παπασταϊκούδης,[*] Πρόδρομος Προδρομίδης,[♦]

Jeremy Watson,[°] Ιωάννης Λέστας[◊]

Η εργασία παρουσιάζει και αναλύει την περίπτωση μιας κεντρικής οντότητας που προσπαθεί να βελτιστοποιήσει την χρησιμότητα των ομάδων ή κοινοτήτων που ευρίσκονται στην περιοχή ασκήσεως των καθηκόντων της, βάσει των προτιμήσεων τους και του υπάρχοντος προϋπολογισμού. Τέτοια σχήματα κεντρικών οντοτήτων και κοινοτήτων μπορούν να θεωρηθούν: μία περιφέρεια και οι περιφερειακές ενότητες της ή οι δήμοι εν αυτή, μία εθνική κυβέρνηση και οι περιφέρειες της χώρας, καθώς και η Ευρωπαϊκή Ένωση και τα κράτη-μέλη της. Για την επίλυση του εν λόγω οικονομικού προβλήματος θα χρειαστεί να το μοντελοποιήσουμε (περιγράψουμε και αποδώσουμε) μαθηματικά και να το ανάγουμε σε ένα πρόβλημα βελτιστοποίησης υπό περιορισμούς.

Στο υπόδειγμα, κάθε κοινότητα έχει τον δικό της στόχο που εκφράζει τις προτεραιότητες/προτιμήσεις των πολιτών της αναφορικά με τα διάφορα αγαθά και υπηρεσίες, εκφρασμένο με την μορφή μίας συνάρτησης χρησιμότητας, καθώς και τον δικό της προϋπολογισμό. Τουλάχιστον ένα από τα επιθυμητά αγαθά ή τις επιθυμητές υπηρεσίες που υπεισέρχονται στην συνάρτηση χρησιμότητας είναι δημόσιο αγαθό ή δημόσια υπηρεσία. Τουτέστιν, είναι ένα αγαθό ή μια υπηρεσία που ωφελεί όλους (δηλ. ωφελεί και άλλα άτομα ή άλλες κοινότητες), καθώς κανείς δεν μπορεί να στερηθεί την ευκαιρία να το/την καταναλώσει, ενώ η κατανάλωση

[*] Επισκέπτης ερευνητής στο ΚΕΠΕ. Μεταδιδακτορικός Ερευνητής Πανεπιστήμιο του Cambridge, Ηνωμένο Βασίλειο. E-mail: ip352@cam.ac.uk

[♦] Ερευνητής Α' Βαθμίδος του ΚΕΠΕ. Αμερικής 11, Αθήνα, 10672. E-mail: pjprodr@kepe.gr. Τηλ. 210-3676412.

[°] Κύριος Λέκτορας, Πανεπιστήμιο του Canterbury, Νέα Ζηλανδία.

[◊] Καθηγητής, Πανεπιστήμιο του Cambridge, Ηνωμένο Βασίλειο.

ενός ατόμου ή μιας κοινότητας δεν επηρεάζει την ευκαιρία ενός άλλου ατόμου ή μιας άλλης κοινότητας να το/την καταναλώσει.

Επίσης, η κεντρική οντότητα γνωρίζει τις προτιμήσεις (συναρτήσεις χρησιμότητας) των επιμέρους κοινοτήτων αναφορικά με τα διάφορα αγαθά και τις διάφορες υπηρεσίες −ιδίως τα δημόσια αγαθά και υπηρεσίες− και αν δεν τις γνωρίζει τις μαθαίνει. Τις διατυπώνουν οι εκπρόσωποι/κυβερνήτες των κοινοτήτων οι οποίοι και τις γνωρίζουν, όταν αιτούνται χρηματοδότηση ή αποδέσμευση της χρηματοδότησης από την κεντρική οντότητα. Οι εκπρόσωποι είναι εκλεγμένοι από τον τοπικό πληθυσμό και ενεργούν για αυτόν. Γνωρίζουν και νοιάζονται για τη συνολική χρησιμότητα των εκλογέων τους και επιδιώκουν να προάγουν την ευημερία τους μέσω των περιορισμένων πόρων που τους διατίθενται.

Ούτως εχόντων το πραγμάτων, στο υπόδειγμα η κεντρική οντότητα έχει να βελτιστοποιήσει −όχι μια συνάρτηση στην οποία η ίδια έχει πλήρη έλεγχο, αλλά− με *αποκεντρωμένο τρόπο* μια συνάρτηση που αποτελείται από ένα άθροισμα επιμέρους συναρτήσεων, κάθε μία εκ των οποίων βελτιστοποιεί τον στόχο μίας τοπικής κοινότητας και η οποία συνάρτηση απαρτίζεται από:

- Κάποιες κοινές (διακοινοτικές) μεταβλητές για δημόσια αγαθά που είναι κοινά ή δημόσιες υπηρεσίες που είναι κοινές σε όλες ή σε κάποιες κοινότητες (*μεταβλητές σύζευξης*, σε τεχνική ορολογία). Λ.χ., έργα σιδηροδρόμων και αυτοκινητοδρόμων που συνδέουν πολλές περιοχές, μεγάλες γέφυρες που συνδέουν γεωγραφικές περιοχές που διαφορετικά δεν γίνεται να συνδεθούν, μεγάλα ηλεκτρικά δίκτυα που μεταφέρουν ενέργεια σε μεγάλες αποστάσεις, μεγάλα νοσοκομεία που εξυπηρετούν δύο ή τρεις περιοχές κ.λπ.
- Κάποιες τοπικές (ενδοκοινοτικές) μεταβλητές για δημόσια αγαθά ή δημόσιες υπηρεσίες που αξιοποιούνται αποκλειστικά από μια κοινότητα. Λ.χ., έργα πρόληψης πλημμυρών και άρδευσης σε μια αγροτική περιοχή που αντιμετωπίζει επίσης προβλήματα πλημμυρών, ή ηλιακά πάνελ και συστήματα ανεμογεννητριών σε μια περιοχή που απομακρύνεται από τα ορυκτά καύσιμα, ή ένα μικρό οδικό δίκτυο και ένα μικρό δίκτυο πρωτοβάθμιας υγειονομικής περίθαλψης σε μια αγροτική περιοχή, κτίρια σχολείων σε άλλη περιοχή κ.λπ.

Υπό αυτό το πρίσμα η υπό βελτιστοποίηση συνάρτηση μπορεί να περιγραφεί ως μια δομή δικτύου που βασίζεται στις διασυνδέσεις των μεταβλητών σύζευξης των διαφόρων στόχων −επιλέγεται αυτή του υπεργραφήματος (*hypergraph*)− και η ζήτηση για τα δημόσια αγαθά και τις δημόσιες υπηρεσίες (για διαδημοτικά/δια-περιφερειακά/διεθνή δημόσια έργα) να διατυπωθεί σε όρους αγοραίων τιμών των αγαθών/υπηρεσιών και των εισοδηματικών ή άλλων περιορισμών με στόχο να μεγιστοποιηθεί η ευημερία των αντίστοιχων πληθυσμών των περιοχών.

Στον βαθμό που η λειτουργία των διακοινοτικών μεταβλητών βελτιώνει την απόδοση των ενδοκοινοτικών μεταβλητών και αντιστρόφως, το πρόβλημα μπορεί να θεωρηθεί ως πρόβλημα ταυτόχρονης πολυδιάστατης βελτιστοποίησης πολλών φορέων με τις μεταβλητές σύζευξης και τις τοπικές μεταβλητές να περιλαμβάνονται στις συναρτήσεις των στόχων. Στην περίπτωση αυτή οι τεχνικές της δικτυοκεντρικής βελτιστοποίησης παρέχουν μια κατάλληλη προσέγγιση λύσης. Χρησιμοποιώντας έναν αλγόριθμο αποσύνθεσης των αρχικών μεταβλητών υπολογίζεται η λύση της ζήτησης για τις μεταβλητές σύζευξης. Όταν υφίσταται μονάχα μια μεταβλητή σύζευξης τότε υπό συγκεκριμένες υποθέσεις η ζήτηση μπορεί να υπολογιστεί αναλυτικά. Όταν υφίστανται πολλαπλές μεταβλητές σύζευξης τότε το πρόβλημα μπορεί να επιλυθεί με την χρήση αριθμητικών μεθόδων καθώς, λόγω της αυξημένης μαθηματικής πολυπλοκότητας του προβλήματος, η παροχή αναλυτικών λύσεων για την ζήτηση δεν είναι εφικτή.

Δεδομένου ότι η βελτιστοποίηση που αφορά σε πολλούς φορείς και δημόσια έργα εθεωρείτο κάποτε αδύνατη (Samuelson, 1954), η ικανότητά μας να την επιλύσουμε είναι ενθαρρυντική και προσφέρει μια αίσθηση προόδου. Ουσιαστικώς, μέσω του υποδείγματος μπορούμε να δούμε μακρύτερα από τις προηγούμενες γενιές οικονομικών αναλυτών και να λύσουμε πρακτικά προβλήματα που σχετίζονται με τον σχεδιασμό της χωρικής κατανομή των δημόσιων αγαθών και υπηρεσιών με βάση τις τοπικές (ή υποπεριφερειακές) προτιμήσεις των τοπικών πληθυσμών.

Με την παρούσα εργασία για συζήτηση το εργαλείο επιστημονικού σχεδιασμού της ενδοπεριφερειακής και περιφερειακής κατανομής πόρων και ανάπτυξης που περιγράφεται στις επόμενες σελίδες τίθεται στον δημόσιο διάλογο και στην διάθεση των ενδιαφερομένων σε μια λειτουργικώς και χωρικώς κατακερματισμένη χώρα όπως η Ελλάδα, να το αξιοποιήσουν ακόμα και ως σημείο εκκίνησης χάριν συγκρίσεων.

***Λέξεις κλειδιά:*** Δικτυοκεντρική βελτιστοποίηση· Αποκεντρωμένη δημοσιονομική· Μεγιστοποίηση της ωφέλειας και συναρτήσεις ζήτησης· Σχεδιασμός περιφερειακής και τοπικής πολιτικής· Δημόσιες υποδομές, αγαθά και υπηρεσίες

# 1 Introduction

In a typical economics utility maximization problem (UMP), one works out how much of each available good or service a consumer will demand (purchase) in order to reach the highest level of satisfaction given his or her preferences and income, time, and other constraints. In essence, one finds the so-called Marshallian demand functions of the goods and services involved by expressing the quantities demanded in terms of prices and the constraints.[1] E.g., Begg et al. (2014: 190-205).

One may think of several such problems: (a) A person or a community maximizing, respectively, his/her or a collective utility function involving the consumption of a number of private goods and services. (b) A social planner maximizing the sum or product of different group or community utility functions with or without a unified procurement system, and separate budgets for each group or community. (c) A social planner or policy coordinator that does the same, with one of the desired goods or services being public, i.e., benefiting all (for no one may be denied the opportunity to consume it, while the consumption of one person or community does not affect the opportunity of another person or community to consume it). (d) Many more. E.g., by Bergstrom and Goodman (1973), King (1986), Bergstrom (1999), Corchón and Dahm (2011), and the sources supplied therein.

All of these problems involve an optimization process. Optimization plays a major role in economics and its wide use has been established in the seminal work of Arrow et al. (1958). The standard assumption of the UMP, is that an agent's utility is described by a single objective function and its respective domain (Mas-Colell et al., 1995). Understandably, an agent's objective may also depend on multiple factors and be described via functions that do not necessarily share the same domain.[2] This introduces a form of decentralization. The summation of these functions constitutes the total objective function of the agent. Often the constituent functions feature common choice variables (coupling variables hereinafter). As a result, the decentralization of the objective factors can be described by a network structure based on the coupling variables' interconnections of the different objectives.

The present paper studies optimization from a public economics viewpoint. Grown from the field of public finance,[3] public economics consider the economics of

[1] There also exists, the so-called Hicksian demand in terms of the prices and utility, that results from the mirror expenditure minimization problem.

[2] To put it differently, an objective function may depend on multiple agents, each of whom optimizes a local objective, rather than having a single central entity controlling everything.

[3] See Desmarais-Tremblay et al. (2023). Public finance, studies finances within a government, so frequently assesses the government's revenues and expenditures (or the

the public sector by looking into government policy through the lens of efficiency and equity in order to improve social welfare, hence frequently rely on microeconomic theory tools. In particular, the paper focuses on government expenditure and assumes that specific budgets are allocated *a priori* to the various local governments by the central government, and local policy makers (agents) are responsible for covering the needs of their respective communities in terms of public goods and services. In the case of municipal districts the local policy makers would be the mayors, and in the case of states the agents would be the governors.

This process can be viewed as a form of decentralized public finance or fiscal policy exercise which leads to a distributed public economics problem, in the sense that multiple mayors or governors will have to distribute their budgets in an optimal way in order to achieve public economic goals in terms of public goods and services that benefit their respective communities. In our setting we will assume that the policy makers of neighboring municipalities or states/provinces/territories act cooperatively in order to achieve the common goals of their regions. This suggests that our theory could also be extended at the country level, and applied to countries acting in a cooperative way in order to achieve certain goals for the benefit of their populations, e.g., the member states of the E.U.

Public finance decentralization or fiscal decentralization is about how central governments empower subnational governments to service their populations (Bahl and Bird, 2018). If the standard three fiscal functions are stabilization, redistribution, and resource allocation, across government levels (Oates, 1972), and the first two are responsibilities of the central government, in our view there is space for improvement through resource allocation via fiscal decentralization. The basic argument in support of fiscal decentralization is that local politicians know people's preferences in their jurisdictions better than does the central government, and, therefore, can better align the provision of public goods and services to those preferences. Fedelino and Ter-Minassian (2010) suggest that there may exist various flaws with fiscal decentralization. In our setting we take the policy makers to be utilitarian social planners elected by the people for the people, i.e., they know and care about the aggregate utility of their respective constituencies, and with the available limited resources each aims to advance the welfare of the population that he or she represents or governs. This is accomplished via the construction and operation of public infrastructure.

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revenues and expenditures of public authorities) and the adjustment of one or the other to achieve desirable effects and avoid undesirable effects.

This public infrastructure can be intra-regional (public goods/services only for the policy maker's region), hence, associated with the specific characteristics of the said region or inter-regional (public goods/services for multiple regions). Examples of intra-regional goods/services are flood prevention and irrigation works for an agricultural region that also faces flood problems, or solar panels and wind turbine systems in a region that moves away from fossil fuels, or a small road network and a small primary health care network in a rural region, or sewage and sanitation systems and school buildings in another region, and so forth. Examples of inter-regional goods/services are rail- and motorways linking multiple regions, long bridges linking geographical areas not connected otherwise, large electrical grids carrying power over long distances, large hospitals servicing two or three regions, etc.

A good number of models that happen to focus on other issues do not concentrate on this aspect of public good or service differentiation. However, in the following pages in order to capture, formulate, and model a policy maker's total objective or aggregate utility (welfare) function we treat: (i) the inter-regional public infrastructure as coupling variables (i.e., as variables that appear in the utility functions of multiple regions), (ii) the intra-regional public infrastructure as local variables in the constituent utility functions, and (iii) the welfare optimization problem as a utility maximization problem.

Under specific assumptions we solve the problem in terms of the Marshallian demand for both the coupling and the local variables. To the extent the presence and operation of interregional (or coupling) variables improves the performance of the intra-regional (or local) variables and vice versa on account of the network effects,[4] the problem can be viewed as a multi-agent, multi-objective optimization problem with coupling and local variables within the objectives. As a result, distributed optimization techniques provide a suitable solution approach.

Distributed optimization can be traced back to Tsitsiklis (1984), and a review on the topic is provided in Yang et al. (2019). The goal in a multi-objective problem is to optimize a global (total) objective, which is usually the sum of the individual objectives of each agent, given their described interconnections, with the use of a graphical structure, e.g., a network. Indeed, networks have been considered and used both in modern economic theory (e.g., Jackson, 2010) and in multi-agent systems with

[4] For instance, (a) solar panels, wind turbine systems and large electrical grids carrying power or (b) a primary health care networks and a large hospital or (c) a small rural road network, an interstate rail- and motor- way and long bridges, etc., may complement each other and jointly improve the economic situation across regions by reducing private costs, raising productivity and making people's lives easier.

economic interpretations (e.g., Cech et al., 2013; Gibson, 2007; Haber, 2014; Eymann, 2001).

Here we make use of a primal decomposition algorithm similar to the one supplied by Papastaikoudis et al. (2024) to solve the distributed optimization problem. We choose the problem's network structure to be a hypergraph[5] in the same fashion as in Samar et al. (2007) and calculate the optimal value of the UMP. That is, we calculate the Marshallian demand solution to the UMP for the coupling variables of the utility functions and find how to allocate inter-regional public works across various regions in a manner that maximizes the welfare of the respective regional populations. In our view this may be extremely useful to a government or an inter-regional coordination committee in charge of territorial development fund allocations.

In particular, in the case of a single coupling variable and under specific assumptions we are able to calculate the Marshallian demand analytically. In the general case of multiple coupling variables in the UMP we cannot provide the analytical solutions of the Marshallian demand due to the increased mathematical complexity of the problem. However, we may calculate and numerically solve the problem. Given that an optimization involving many parties and public goods/services was once considered impossible to solve (Samuelson, 1954), our ability to solve it is encouraging and offers a sense of progress. In essence, through this paper we can see further than we could see before, and solve practical problems associated with the spatial distribution of public goods and services based on subregional preferences.

Our proposed setting does not deviate from standard economic theory. We assume that an input vector (choice variables) under a mechanism (utility function) corresponds to specific levels of utility. The mechanism is separable and can be written as the summation of different mechanisms (objective functions) with couplings among the input variables of these mechanisms. This assumption does not violate any standard economic principles of utility theory (Samuelson, 1938; Debreu, 1959a,b; Uzawa, 1960; Lancaster, 1966) and it may be viewed as an extension. Literature regarding the multi objective optimization problem can be found in Hamel and Wang (2017), Zhao et al. (2017), Evren (2014). Our proposed setting and solution approach may also be

[5] Hypergraphs were first introduced in Berge (1973) as a generalization of graphs since they allow more than two nodes to be linked in the same edge (hyperedge). The various advantages of hypergraph communication are presented in Heintz et al. (2019), Wolf et al. (2016), Heintz and Chandra (2014). In our view a hypergraph turns out to be more suitable to describe the information structure of the coupling variables compared to a graph that uses pairwise interconnections that do not have any particular economic interpretation in the model.

extended to other static optimization economic problems such as the profit maximization problem.

The rest of the paper is organized as follows: Section 2 introduces the reader to mathematical tools borrowed from non-linear control theory (Khalil, 2002) and hypergraph theory (Vitaly, 2009). Section 3 sets up the hypergraph distributed optimization problem and the primal decomposition algorithm along with a stability analysis. Section 4 outlines the centralized UMP for the case of logarithmic Cobb-Douglas public sector objective functions and outlines its respective decentralized version in the case of a single coupling variable, as well as in the case of multiple coupling variables. Examples are provided as we build up the theory. Section 5 concludes.

# 2 Preliminaries

## 2.1 Non Linear Control Theory

We study the following continuous, time invariant system,

$$\dot{x}(t) = f(x(t)) \tag{1}$$

with $x(t) \in \mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}^n$ being continuous. A function $V : \mathbb{R}^n \to \mathbb{R}$ for which the following relationship $||x|| \to \infty \Rightarrow V(x) \to \infty$ holds is called radially unbounded. By $\dot{V} : \mathbb{R}^n \to \mathbb{R}$ is denoted the Lie derivative of $V$ which is expressed by the following formula:

$$\dot{V}(x(t)) = \nabla V(x(t))^T \cdot \dot{x}(t) = \nabla V(x(t))^T \cdot f(x(t)).$$

**Theorem 1.** *Let $x^*$ be an equilibrium point of (1). If $V : \mathbb{R}^n \to \mathbb{R}$ is positive definite, radially unbounded and $\dot{V}(x) < 0, \forall\ x \neq x^*$ then $x^*$ is globally asymptotically stable and $V$ is a valid Lyapunov function for (1).*

## 2.2 Hypergraphs

A hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{v_1, ..., v_n\}$ is the finite set of nodes and $\mathcal{E} = \{\mathcal{E}_1, ..., \mathcal{E}_m\}$ is the corresponding set of hyperedges can be viewed as a generalization of a graph in the sense that each hyperedge can join any number of nodes and not just two. The order of a hypergraph $\mathcal{H}$ denoted by $|\mathcal{V}|$ is the total number of nodes while its size denoted by $|\mathcal{E}|$ is the total number of hyperedges. For a hypergraph $\mathcal{H}$, the incidence matrix, denoted by $E$ is a $|\mathcal{V}| \times |\mathcal{E}|$ matrix whose $(i, j)$-th entry is defined as:

$$E = \begin{cases} 1, & v_i \in \mathcal{E}_j \\ 0, & \text{otherwise.} \end{cases}$$

# 3 Hypergraph Distributed Optimization

## 3.1 Problem Formulation

Assuming that we have $K$ different interconnected subsystems where each subsystem has an objective function and their interconnections are represented via their objectives, the optimization of their objectives leads to the following distributed optimization problem,

$$\begin{aligned} \max_{[x_1,y_1,...,x_K,y_K,z]} \quad & \sum_{i=1}^{K} F_i(x_i, y_i) \\ \text{s.t. } & (x_i, y_i) \in \mathcal{C}_i \\ & x_i = E_i z, \ i = 1, ..., K. \end{aligned} \tag{2}$$

where

- $F_i : \mathbb{R}^{(p_i+m_i)} \to \mathbb{R}$ is the objective function of $i$th subsystem and is considered to be strictly concave, continuously differentiable with its gradient $\nabla F_i$ being locally Lipschitz.
- Vectors $x_i \in \mathbb{R}^{p_i}, \forall\ 1 \leq i \leq K$ denote the variables of the subsystems which we assume are coupling (i.e. their components appear in the variables of other subsystems as well).
- Vectors $y_i \in \mathbb{R}^{m_i}, \forall\ 1 \leq i \leq K$ denote the variables of the subsystems which we assume are local (i.e. they appear in only one subsystem).
- $\mathcal{C}_i$ is a feasible set for subsystem $i$, described by linear equalities and convex inequalities.
- Vector $z \in \mathbb{R}^N$ gives the respective common values of the $N$ different groups of coupling variable components.
- The relationship $x_i = E_i z$ allocates the variable components of $i$th subsystem to their respective common values and $E_i$ is a $p_i \times N$ matrix whose $(l, j)$-th entry is given by
$$E_i^{lj} = \begin{cases} 1, \text{ if } x_i^l = z_j, \ \forall\ 1 \leq l \leq p_i \\ 0, \quad \text{otherwise} \end{cases} \tag{3}$$
with $x_i^l$ denoting the $l$th component of variable $x_i$.

In order to represent the network effects that the coupling variables create among the different objective functions we will use a graphical structure and we choose it to be a hypergraph since there may be more than two coupling variables that describe a common value. We consider the following hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ where,

- the set of nodes $\mathcal{V}$ is partitioned into $\mathcal{V} = \{\mathcal{V}_1, ..., \mathcal{V}_K\}$ with each node in subset $\mathcal{V}_i$ being associated with a component of variable $x_i$, $\forall\ 1 \leq i \leq K$.
- each hyperedge $\mathcal{E}_j$ is associated with the $j$th component of vector $z$, i.e, the hyperedge set $\mathcal{E}$ is describes the couplings of different variable components.

As a result, for the hypergraph $\mathcal{H}$ we have,

$$|\mathcal{V}| = p_1 + ... + p_K = p, |\mathcal{E}| = N, E = \begin{bmatrix} E_1 \\ \vdots \\ E_K \end{bmatrix},$$

$$D_V = I_{p\times p} \text{ and } D_E = \text{diag}\{|\mathcal{E}_1|, ..., |\mathcal{E}_N|\}.$$

Hence, by $x_i^j$ we denote the coupling variable of the $i$th subsystem that belongs to the $j$th group of coupling variables for $i = 1, ..., K$ and $j = 1, ..., N$. The relationship $x_i = E_i z, \forall\ 1 \leq i \leq K$ can also be written as $x = Ez$ where $x = (x_1, ..., x_K) \in \mathbb{R}^p$.

**Remark 1.** *We assume that the structure of the objective functions and their respective coupling variables would result to a connected hypergraph in the sense that there would be a connection path among every two nodes of the hypergraph.*

For the better understanding of graphical/network setting we present the following example.

**Example 1.** *Assuming that we have four subsystems as they are depicted in Figure 1 with the following sum of their respective objective functions to be*

$$f_1(x_1^1) + f_2(x_2^1) + f_3(x_3^1, x_3^2) + f_4(x_4^2)$$

*we notice that there are two pairs of coupling variable components among the subsystems. The nodes associated with variables $\{x_1^1, x_2^1, x_3^1\}$ are attached to hyperedge $\mathcal{E}_1$*

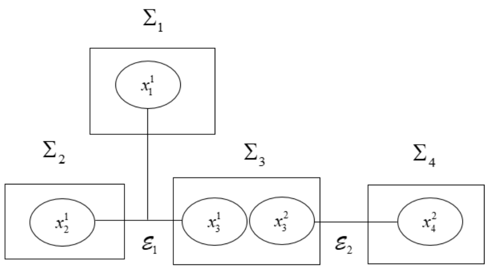


**Fig. 1** Hypergraph Communication

*while the nodes associated with variables $\{x_3^2, x_4^2\}$ are attached to hyperedge $\mathcal{E}_2$. We also have that $|\mathcal{V}| = 5, |\mathcal{E}| = 2, D_V = I_{5\times 5}$,*

$$D_E = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} \text{ and } E = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = \begin{pmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 1 \end{bmatrix} \end{pmatrix}$$

*respectively.*

### 3.2 Primal Decomposition

We let $f_i(x_i)$ denote the optimal value for the local variable of the $i$th subproblem,

$$\max_{y_i} F_i(x_i, y_i)$$

$$\text{s.t. } (x_i, y_i) \in \mathcal{C}_i \tag{4}$$

and we express variable $y_i$ in terms of $x_i$ by solving the optimization problem (4). Functions $f_i, i = 1, ..., K$ are strictly concave as well since maximization with respect to a variable of a function preserves strict concavity. As a result, the Lagrangian of (2) is given by:

$$\mathcal{L}(x, z, v) = \sum_{i=1}^{K} f_i(x_i) - v^T(x - Ez) \tag{5}$$

which can also be written as

$$\mathcal{L}(x, z, v) = \sum_{i=1}^{K} (f_i(x_i) - v_i^T x_i) + v^T Ez$$

where $v_i$ corresponds to $i$th subsystem and is a subvector of Lagrange multiplier $v$ associated with $x = Ez$. The optimality conditions are:

$$\frac{\partial L}{\partial x} = 0 \Rightarrow \nabla f_i = v_i \text{ (subsystems interconnections)} \tag{6a}$$

$$\frac{\partial L}{\partial v} = 0 \Rightarrow x = Ez \text{ (primal feasibility)} \tag{6b}$$

$$\frac{\partial L}{\partial z} = 0 \Rightarrow E^T v = 0 \text{ (dual feasibility).} \tag{6c}$$

**Remark 2.** *Throughout this work we will specifically study cases where the resulting functions $f_i$ from (4) are differentiable but this is not the case in general.*

The original problem (2) is equivalent to the primal problem,

$$\max_{z} f = \sum_{i=1}^{K} f_i(E_i z) \tag{7}$$

To find a gradient of $f$, we calculate $\nabla f_i, i = 1, ..., K$ where functions $\nabla f_i, i = 1, ..., K$ are strictly monotone. In primal decomposition, at each iteration we fix the vector $z$ of common values and the coupling variables as $x_i = E_i z$. We then have

$$\nabla f(z) = \sum_{i=1}^{K} E_i^T \nabla f_i(x_i).$$

For the solution of (7) we will use the following gradient method

$$\dot{z} = -\nabla f(z) = -\sum_{i=1}^{K} E_i^T \nabla f_i(x_i) \tag{8}$$

**Remark 3.** *It is important to note that this distributed optimization setting can be viewed as a multiple consensus problem, a consensus value must achieved for each hyperedge, i.e. the coupling variables components attached to each hyperedge will be represented by the respective variable component of vector $z$ asymptotically.*

**Theorem 2.** *Let $x^*$ be an equilibrium point of the dynamical system (8) then $x^*$ is a solution to the optimization problem (2).*

*Proof.* In order to prove the theorem we decompose (8) in the following dynamical system

$$x_i(t) = E_i z(t) \tag{9a}$$

$$v_i(t) = \nabla f_i(x_i(t)) \tag{9b}$$

$$w(t) = \sum_{i=1}^{K} E_i^T v_i(t) \tag{9c}$$

$$\dot{z}(t) = w(t) \tag{9d}$$

We find the equilibrium point of the dynamical system (8) from $\dot{z}(t) = 0 \Rightarrow w(t) = 0 \Rightarrow \sum_{i=1}^{K} E_i^T v_i(t) = \sum_{i=1}^{K} E_i^T \nabla f_i(x_i(t)) = 0$ which is equal to the last of the optimality conditions in (6$c$). The rest of the optimality conditions are trivially satisfied from the rest equations of the decomposed form of (8) given by the dynamical system (9$a$)-(9$d$) and as a result, the equilibrium point of (8) is a solution of the optimization problem (2).

□

**Theorem 3.** *If $z^*$ is an equilibrium point for the dynamical system (8) then $z^*$ is globally asymptotically stable.*

*Proof.* We choose for the dynamical system (8) as a Lyapunov candidate the function $V(t) = \frac{1}{2}||z(t) - z^*||_2^2$ where $V(t) : \mathbb{R}^N \to \mathbb{R}$ is radially unbounded. The Lie derivative of the Lyapunov function is

$$\begin{aligned}
\dot{V}(t) &= \dot{z}(t)^T (z(t) - z^*) \\
&= (E^T \nabla f(x(t)))^T (z(t) - z^*) \\
&= (E^T \nabla f(x(t)) - E^T \nabla f(x^*))^T (z(t) - z^*) \\
&= (\nabla f^T(x(t)) - \nabla f^T(x^*))(Ez(t) - Ez^*) \\
&= (\nabla f^T(x(t)) - \nabla f^T(x^*))(x(t) - x^*) < 0, \ \forall \ x(t) \neq x^*.
\end{aligned}$$

since $\nabla f$ is strictly monotone and as a result, $\dot{V}(t) < 0$ for $x(t) \neq x^*$. We have used in the proof that $E^T \nabla f(x^*) = 0$ and $x = Ez$. From the above we conclude that the equilibrium point $z = z^*$ is globally asymptotically stable for the dynamics (8). $\square$

# 4 Utility Maximization Problem

The utility maximization problem in its economic version is usually described as the problem of a single individual who wants to answer the question of how much to choose of each available good or service to consume, taking into account his budget constraint, the prices of the goods/services and his own preferences which are expressed in terms of a utility function.

In this work we will use the techniques of the utility maximization problem in order to answer to a similar question but from the scope of an agent or a group of agents who are policy makers of specific regions, they act in a cooperative way and they want to allocate public goods across their regions, under specific budgets and costs in a way that optimizes the welfare of the populations of the regions under consideration. We assume that there are two types of public goods, the intra-regional goods that have to do with the welfare of a specific region and the inter-regional goods that have to do with the welfare of multiple regions. Both types of goods will be included in the utility function of each region and the inter-regional goods will create network effects among the utility functions of the regions making distributed optimization techniques as the most suitable for the solution of the respective optimization problem.

We will start this section by discussing the simplest possible case, that of a single policy maker agent who is trying to optimize the welfare of its population expressed in terms of a utility function by allocating the local public goods of his region for the given budget and costs. We call this case centralized since there are no inter-regional public goods and as a result, no network effects created with other regions.

## 4.1 Centralized Case

The problem of local public goods/services allocation that a policy maker of a single region faces with limited budget resources is the following:

$$\max_{y \geq 0} u(y)$$

$$\text{s.t. } p \cdot y \leq w. \tag{11}$$

where $u : \mathbb{R}^n \rightarrow \mathbb{R}_+$ is the utility function of the region under consideration, vector $y = (y_1, y_2, ..., y_n) \in \mathbb{R}^n_+$ describes the $n$ local public goods/services while $p = (p_1, p_2, ..., p_n) \in \mathbb{R}^n_+$ represents the respective price costs for the construction/operation of a unit of the respective public good/service and by $w \in \mathbb{R}_+$ we denote the respective budget wealth of the region. Both the price costs and the budget wealth are parameters. The optimum solution of the policy makers utility maximization problem in terms of its parameters is called Marshallian demand and is of the form

$$y^*(p, w) = (y_1^*(p, w), y_2^*(p, w), ..., y_n^*(p, w)). \tag{12}$$

The Marshallian demand in our setting shows the demand for each public good/service given the specific needs of the population of the region which are captured in the formulation of the region's utility function and provides an indication to the policy maker of how he should partition the region's budget for each local public good/service. Also, the Marshallian demand for each public good/service provides a hint of the quality of the future construction/service, i.e., a larger portion of the budget indicates a project of higher standards while a smaller portion of the budget indicates a project of not so much importance, e.g., the construction of a bigger or smaller school or hospital or road depending on the needs of the region as these are captured in the Marshallian demand. In other words it provides a measure of quality classification for the future public good/service provided.

One of the most commonly used utility function is the Cobb-Douglas utility function,

$$u(y_1, ..., y_n) = \prod_{i=1}^{n} y_i^{a_i}, \ a_i > 0 \ \forall \ i,$$

which we believe is a suitable choice of utility function due to its many attractive properties such as, its algebraic tractability and its overall fairly good approximation of the utility gaining process among others. Due to the technicalities that can be caused by the exponential form of the function we will use instead a logarithmic Cobb-Douglas function,

$$u(y_1, ..., y_n) = \sum_{i=1}^{n} a_i \log y_i, \ a_i > 0 \ \forall \ i,$$

a choice that will also facilitate us later for the decentralized case. It is important to note that the logarithm is an increasing function and represents the same underlying preferences as the Cobb-Douglas utility function. We interpret the parameters $a_i$ of the utility function as sensitivity factors of the population to the $i$th public good/service, higher $a_i$ suggests higher desire of the population for the suggested public good/service while lower $a_i$ suggest of a relatively small interest in the respective public good/service. The centralized UMP for the logarithmic Cobb-Douglas utility function is the following

$$\begin{aligned} \max_{y_i} \quad & \sum_{i=1}^{n} a_i \log y_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} p_i \cdot y_i \leq w \\ & y_i > 0, \ \forall \ 1 \leq i \leq n. \end{aligned} \tag{13}$$

**Remark 4.** *Of course other forms of utility functions can be used depending of the specific characteristics of the areas under consideration.*

**Theorem 4.** *The optimum solution (Marshallian demand) of (13) for each public good/service $y_i$ is given by the following formula*

$$y_i^* = \frac{a_i w}{p_i \sum_{i=1}^{n} a_i}. \tag{14}$$

*Proof.* Since the budget constraint is $w > 0$ the positivity constraints on $y_i$ will always be satisfied. The budget inequality constraint binds by Walras' law

$$w = \sum_{i=1}^{n} p_i \cdot y_i,$$

and as a result, we can form the Lagrangian of problem (13) to be

$$L = \sum_{i=1}^{n} a_i \log y_i + \lambda(w - \sum_{i=1}^{n} p_i \cdot y_i)$$

giving first-order conditions

$$\frac{a_i}{y_i} = \lambda p_i, \ \forall\ 1 \leq i \leq n. \tag{15}$$

Using the expressions in (15) we could write all the variables in terms of variable $y_1$:

$$y_2 = \frac{p_1 a_2}{p_2 a_1} y_1, ..., y_n = \frac{p_1 a_n}{p_n a_1} y_1. \tag{16}$$

Combining the expresions of all variables in terms of $y_1$ and the budget constraint we have,

$$w = p_1 y_1 + \sum_{i=2}^{n} p_i \cdot \frac{p_1 a_i}{p_i a_1} y_1$$

and we find the Marshallian demand functions to be:

$$y_i^* = \frac{a_i w}{p_i \sum\limits_{i=1}^{n} a_i}, \ \forall\ 1 \leq i \leq n. \tag{17}$$

□

We note that the quantity $\frac{a_i}{p_i \sum_{i=1}^{n} a_i}$ for our optimal solution $y_i^*$ in (17) suggests the portion of the budget $w$ that will be allocated for the public good/service $y_i$ as discussed in the introductory part of this subsection.

## 4.2 Decentralized Case

We call the case of multiple regions where there are both types of public goods/services, i.e., intra and inter-regional, decentralized since it is conducted in a decentralized finance setting where in the decisions regarding the inter-regional public goods/services are getting involved multiple policy makers in a cooperative way and use the respective budget wealth of their regions in order for the respective public infrastructure to be completed. The cooperation of the policy makers is captured in the total objective function which is expressed as the summation of the utility functions of all the regions under consideration. For the case of multiple regions with a single inter-regional public good/service the policy makers of the $K$ different regions face the following distributed optimization problem.

$$\begin{aligned}
\max_{x_1,y_1,\dots,x_K,y_K,z} \quad & F_1 + \dots + F_K \\
\text{s.t.} \quad & p_1 x_1 + \sum_{j=1}^{n_1} p_1^j y_1^j \leq w_1 \\
& \vdots \\
& p_K x_K + \sum_{j=1}^{n_K} p_K^j y_K^j \leq w_K \\
& x = Ez \\
& x_1, y_1, \dots, x_K, y_K \geq 0.
\end{aligned} \tag{18}$$

where the $l$th region's utility function is logarithmic Cobb Douglas with the following analytical expression

$$F_l = a_l \log x_l + \sum_{j=1}^{n_l} b_l^j \log y_l^j, \ \forall \, 1 \leq l \leq K. \tag{19}$$

The scalar variables $x_1, \dots, x_K$ are the coupling variables of the utility functions that represent the single inter-regional public good/service, i.e., $x_1 = \dots = x_K$ while $y_1, \dots, y_K$ are the vectors of local variables of the utility functions where $y_l = (y_l^1, \dots, y_l^{n_l}), \ \forall \, 1 \leq l \leq K$ that represent the intra-regional public goods/services of the $l$th region. By $a_l$ we denote the respective parameter of the coupling variable $x_l$ for the $l$th utility factor function that denotes the sensitivity of the $l$th population to the respective inter-regional public good/service. Similarly $b_l^j$ are the respective sensitivity parameters of the $l$th population to the intra-regional public goods/services of their region $y_l^j$ with $n_l$ being the total number of public goods/services for the $l$th region. By $w_l, \forall \, 1 \leq l \leq K$ we denote the total budget of $l$th region. The total budget $w$ for all regions and both types of public goods/services satisfies the following relationship $w = \sum_{l=1}^{K} w_l$. Regarding the parameters of the utility factor functions the following relationship holds

$$a_l + \sum_{j=1}^{m_l} b_l^j > 1, \ \forall \, 1 \leq l \leq K. \tag{20}$$

which means that the utility functions of the regions under consideration have increasing returns to scale something that it is natural for vast inter-regional public goods. From a mathematical perspective the parameters in (20) also ensure the strict concavity of (19). The cost price vector per unit of public good/service for the $l$th region is given by $p^l = (p_l, p_l^1, \dots, p_l^{n_l})$ and for the cost price of the coupling variable we have $p_1 = ... = p_K = p$ where $p$ is the common cost price value. Since we have a single coupling variable, $z$ is a scalar and describes the common value of the coupling variables, i.e., $z = x_1 = ... = x_K$. For the incidence matrix of the hypergraph $\mathcal{H} = \{\mathcal{V}, \mathcal{E}\}$ that assigns the coupling variables to their respective common values we have $E = \mathbf{1}_K$, i.e., the incidence matrix is equal to a vector of ones of dimension $K$.

**Theorem 5.** *The Marshallian demand of the coupling variable $z = x_1 = ... = x_K$ in (18) satisfies the equation*

$$\frac{\sum_{l=1}^{K} a_l}{z} = \sum_{l=1}^{K} \frac{p \cdot \sum_{j=1}^{m_l} b_l^j}{w_l - pz} \tag{21}$$

*and in the case that the budget is distributed equally among all the regions, i.e., $w_l = \frac{w}{K}$, $\forall\ 1 \leq l \leq K$ then equation (21) has an analytical solution with respect to $z$ given by the formula*

$$z = \frac{w \cdot \left(\sum_{l=1}^{K} a_i\right)}{K \cdot p \cdot \left(\sum_{l=1}^{K} [a_l + \sum_{j=1}^{m_l} b_l^j]\right)}. \tag{22}$$

*Proof.* We choose to express all the local variables of each utility function in terms of their respective first local variable and then all the first local variables in terms of the coupling variable respectively. By applying the first order conditions to each subproblem independently in a similar way as in (16) we have the following relationships for the $l$th utility factor function,

$$y_l^j = \frac{b_l^j p_l^1}{b_l^1 p_l^j} \cdot y_l^1,\ \forall\ 1 \leq l \leq K,\ 2 \leq j \leq m_l. \tag{23}$$

Using the Walras' law for the budget constraints of the $l$th utility factor function we have

$$\begin{aligned} p_l^1 y_l^1 + \sum_{j=2}^{m_l} p_l^j y_l^j &= w_l - p_l x_l \\ p_l^1 y_l^1 + \sum_{j=2}^{m_l} p_l^j \frac{b_l^j p_l^1}{b_l^1 p_l^j} \cdot y_l^1 &= w_l - p_l x_l \\ p_l^1 y_l^1 \left(1 + \sum_{j=2}^{m_l} \frac{b_l^j}{b_l^1}\right) &= w_l - p_l x_l \\ y_l^1 &= \frac{b_l^1 (w_l - p_l x_l)}{p_l^1 \left(\sum_{j=1}^{m_l} b_l^j\right)} \end{aligned} \tag{24}$$

Combining (24) and (23) we have that

$$y_l^j = \frac{b_l^j(w_l - p_l x_l)}{p_l^j \left(\sum\limits_{j=1}^{m_l} b_l^j\right)}, \ \forall\ 1 \le l \le K,\ 1 \le j \le m_l. \tag{25}$$

We can now write the utility function $F_l$ by expressing the local variables of the function in terms of the coupling variable $x_l$ as

$$f_l(x_l) = a_l \log(x_l) + \sum_{j=1}^{m_l} b_l^j \log\left[\frac{b_l^j(w_l - p_l x_l)}{p_l^j \left(\sum\limits_{j=1}^{m_l} b_l^j\right)}\right] \ \forall\ 1 \le l \le K. \tag{26}$$

From the primal decomposition algorithm (9$a$)-(9$d$) we have $\dot{z} = \sum\limits_{l=1}^{K} E_l \nabla f_l(x_l)$ where $E_l = 1,\ \forall\ 1 \le l \le K, z = x_1 = ... = x_K$ and $p = p_1 = ... = p_K$. As a result,

$$\dot{z} = \frac{\sum\limits_{l=1}^{K} a_l}{z} - \sum_{l=1}^{K} \frac{p \cdot \sum\limits_{j=1}^{m_l} b_l^j}{w_l - pz}. \tag{27}$$

The equilibrium point of (27) results to the relationship

$$\frac{\sum\limits_{l=1}^{K} a_l}{z} = \sum_{l=1}^{K} \frac{p \cdot \sum\limits_{j=1}^{m_l} b_l^j}{w_l - pz}. \tag{28}$$

The solution of equation (28) in terms of $z = x_1 = ... = x_K$ is the Marshallian demand for the coupling variable. In the case that the budget is distributed equally among all the regions, i.e., $w_l = \frac{w}{K},\ \forall\ 1 \le l \le K$ then equation (28) has an analytical solution with respect to $z$, i.e.,

$$\frac{\sum\limits_{l=1}^{K} a_l}{z} = \frac{p \cdot \sum\limits_{l=1}^{K} \left[\sum\limits_{j=1}^{m_l} b_l^j\right]}{\frac{w}{K} - pz} \Rightarrow$$

$$\frac{\sum\limits_{l=1}^{K} a_l}{z} = \frac{K \cdot p \cdot \sum\limits_{l=1}^{K} \left[\sum\limits_{j=1}^{m_l} b_l^j\right]}{w - K \cdot p \cdot z} \Rightarrow$$

$$w \cdot \sum_{l=1}^{K} a_l - K \cdot p \cdot z \cdot \sum_{l=1}^{K} a_l = K \cdot p \cdot z \cdot \sum_{l=1}^{K} \left[\sum_{j=1}^{m_l} b_l^j\right] \Rightarrow$$

$$K \cdot p \cdot z \cdot \left( \sum_{l=1}^{K} \left[ a_l + \sum_{j=1}^{m_l} b_l^j \right] \right) = w \cdot \sum_{l=1}^{K} a_l \Rightarrow$$

$$z = \frac{w \cdot \sum_{l=1}^{K} a_i}{K \cdot p \cdot \left( \sum_{l=1}^{K} \left[ a_l + \sum_{j=1}^{m_l} b_l^j \right] \right)}.$$

By combining the Marshallian demand of the coupling variable $z = x_1 = ... = x_K$ and the relationship (25) we can find the Marshallian demand of all the local variables for every utility function. □

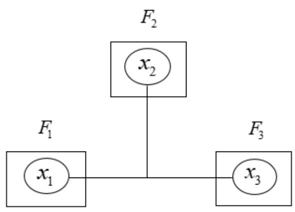


**Fig. 2** Three regions with a single inter-regional public good.

**Example 2.** *In Figure 2 we have three regions with a single inter-regional public good/service with their respective logarithmic Cobb-Douglas utility functions $F_1, F_2$ and $F_3$ to be of the form*

$$F_1 = a_1 logx_1 + b_1^1 logy_1^1$$
$$F_2 = a_2 logx_2 + b_2^1 logy_2^1 + b_2^2 logy_2^2$$
$$F_3 = a_3 logx_3 + b_3^1 logy_3^1$$

*where the variables $x_1, x_2, x_3$ are the coupling variables representing the inter-regional public good/service while $y_1^1, y_2^1, y_2^2, y_3^1$ are the local variables representing the intra-regional public goods/services with their parameters to satisfy the following relationships,$a_1 + b_1^1 \geq 1, a_2 + b_2^1 + b_2^2 \geq 1$ and $a_3 + b_3^1 \geq 1$. Given the cost price vectors $p^1 = (p_1, p_1^1), p^2 = (p_2, p_2^1, p_2^2), p^3 = (p_3, p_3^1)$ and the budget constraints $w_1, w_2, w_3$ for the respective regions we have the following decentralized UMP,*

$$\max_{x,y} F_1 + F_2 + F_3$$

$$\begin{aligned} s.t. \quad & p_1 x_1 + p_1^1 y_1^1 \leq w_1 \\ s.t. \quad & p_2 x_2 + p_2^1 y_2^1 + p_2^2 y_2^2 \leq w_2 \\ s.t. \quad & p_3 x_3 + p_3^1 y_3^1 \leq w_3 \\ & x = Ez \\ & x_1, x_2, x_3, y_1^1, y_2^1, y_2^2, y_3^1 \geq 0 \end{aligned} \tag{29}$$

*where* $x = [x_1, x_2, x_3]^T \in \mathbb{R}^3_+, y = [y_1^1, y_2^1, y_2^2, y_3^1]^T \in \mathbb{R}^4_+, E = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ *and* $z \in \mathbb{R}_+$. *For the solution of* (29) *we will make use of the primal decomposition algorithm* (9a)-(9d). *We will express the local variables of each utility function in terms of the coupling variable. The first and third utility functions have exactly one local variable and with the use of Walras' law for the budget constraints we have* $y_1^1 = \frac{w_1 - p_1 x_1}{p_1^1}$ *and* $y_3^1 = \frac{w_3 - p_3 x_3}{p_3^1}$. *For the second utility factor function we express* $y_2^2$ *in terms of* $y_2^1$ *as* $y_2^2 = \frac{p_2^1 b_2^2 y_2^1}{p_2^2 b_2^1}$ *and from Walras' law we have* $y_2^1 = \frac{b_2^1(w_2 - p_2 x_2)}{p_2^1(b_2^1 + b_2^2)}$ *and* $y_2^2 = \frac{b_2^2(w_2 - p_2 x_2)}{p_2^2(b_2^1 + b_2^2)}$. *The utility function in terms of their coupling variable are given by*

$$f_1(x_1) = a_1 log(x_1) + b_1^1 log\left(\frac{w_1 - p_1 x_1}{p_1^1}\right),$$

$$f_2(x_2) = a_2 log(x_2) + b_2^1 log\left(\frac{b_2^1(w_2 - p_2 x_2)}{p_2^1(b_2^1 + b_2^2)}\right) + b_2^2 log\left(\frac{b_2^2(w_2 - p_2 x_2)}{p_2^2(b_2^1 + b_2^2)}\right),$$

$$f_3(x_3) = a_3 log(x_3) + b_3^1 log\left(\frac{w_3 - p_3 x_3}{p_3^1}\right).$$

*From the primal decomposition algorithm* (9a)-(9d) *we have* $\dot{z} = \sum_{l=1}^{3} E_l \nabla f_l(x_l)$ *where* $E_l = 1, \ \forall \ 1 \leq l \leq 3, z = x_1 = x_2 = x_3$ *and* $p = p_1 = p_2 = p_3$. *As a result,*

$$\dot{z} = \frac{\sum_{l=1}^{3} a_l}{z} - \sum_{l=1}^{3} \frac{p \cdot \sum_{j=1}^{m_l} b_l^j}{w_l - pz}.$$

*The equilibrium point of this dynamical system results to the solution of the following equation in terms of* $z$,

$$\frac{a_1 + a_2 + a_3}{z} = \frac{p \cdot b_1^1}{w_1 - pz} + \frac{p \cdot (b_2^1 + b_2^2)}{w_2 - pz} + \frac{p \cdot b_3^1}{w_3 - pz}.$$

*The solution of this equation is the Marshallian demand for the coupling variables* $z = x_1 = x_2 = x_3$. *In the case that the budget is distributed equally among the three regions, i.e.,* $w_1 = w_2 = w_3 = \frac{w}{3}$ *then the above equation has an analytical solution with respect to* $z$ *given by the formula*

$$z = \frac{w \cdot (a_1 + a_2 + a_3)}{3 \cdot p \cdot (a_1 + b_1^1 + a_2 + b_2^1 + b_2^2 + a_3 + b_3^1)}. \quad (30)$$

*By using the following parameters for the first region* $a_1 = 1.6, b_1^1 = 1$, *for the second region* $a_2 = 1.5, b_2^1 = 0.4, b_2^2 = 0.6$, *for the third region* $a_3 = 1.7, b_3^1 = 1$, *with the respective prices* $p = p_1 = p_2 = p_3 = 2, p_1^1 = 0.7, p_2^1 = 0.5, p_2^2 = 0.9, p_3^1 = 0.8$ *and the respective budgets* $w_1 = 25, w_2 = 30, w_3 = 20$, *we get the asymptotic value of the coupling variable to be*

$$z = x_1 = x_2 = x_3 = 5.875$$

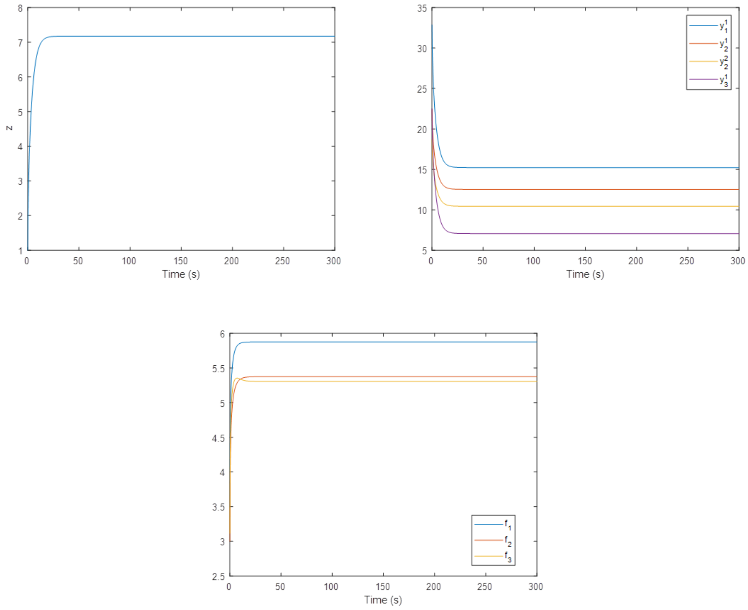


**Fig. 3** (a) Coupling Variables (b) Local Variables (c) Utility Functions

*while the asymptotic values of the local variables are*

$$y_1^1 = 15.22, y_2^1 = 15.523, y_2^2 = 10.436, y_3^1 = 7.068$$

*which results to the following utility optimum values*

$$f_1 = 5.875, f_2 = 5.374, f_3 = 5.305$$

*with the total utility optimal value to be*

$$f = f_1 + f_2 + f_3 = 16.554$$

*In Figure 3 we present the convergence processes of the coupling variable, the local variables and the utility functions for the decentralized case with equal budgets.*

## 4.3 Generalized Case with Multiple Coupling Variables

The generalized version of the decentralized case with multiple inter-regional public goods/services makes the policy makers of the $K$ different regions under consideration to face the following distributed UMP problem.

$$\max_{x_1, y_1, \ldots, x_K, y_K} F_1 + \ldots + F_K$$

$$\text{s.t.} \quad \sum_{i=1}^{m_1} x_1^i p_1^i + \sum_{j=1}^{n_1} y_1^j p_1^{'j} \leq w_1$$

$$\vdots$$

$$\text{s.t.} \quad \sum_{i=1}^{m_K} x_K^i p_K^i + \sum_{j=1}^{n_K} y_K^j p_K^{'j} \leq w_K$$

$$x = Ez$$

$$x_1, y_1, ..., x_K, y_K \geq 0 \tag{31}$$

where vector $z$ describes the common values of the coupling variables and $E$ is the incidence matrix of the hypergraph. An analytical expression of the $l$th utility function is provided below

$$F_l = \sum_{i=1}^{m_l} a_l^i \log x_l^i + \sum_{j=1}^{n_l} b_l^j \log y_l^j, \ \forall\ 1 \leq l \leq K. \tag{32}$$

where our economic narrative remains similar as with the previous case with a single coupling variable with the only difference here to be the existence of multiple coupling variables. We have that:

- $a_l^i$ are the respective parameters of the coupling variables $x_l^i$ and $m_l$ is the total number of coupling variables $\forall\ 1 \leq l \leq K$.
- $b_l^j$ are the respective parameters of the local variables $y_l^j$ and $n_l$ is the total number of local variables for the $\forall\ 1 \leq l \leq K$ and
- $\sum\limits_{i=1}^{m_l} a_l^i + \sum\limits_{j=1}^{n_l} b_l^j > 1, \ \forall\ 1 \leq l \leq K.$

The cost price vector of the $l$th region is

$$p_l = (p_1, ..., p_{m_l}, p_l^{'1}, ..., p_l^{'n_l})$$

where by $p$ and $p'$ we denote the prices for the coupling and local variables respectively. For the total budget, we have,$w = \sum\limits_{l=1}^{K} w_l$.

**Remark 5.** *For the generalized case an analytical expression of the Marshallian demand of the coupling variables for a utility function with multiple coupling variables would be extremely difficult, if not impossible, due to the creation of couplings among the coupling variables via the local variables. In order to overcome this problem for the $l$th utility function with multiple coupling variables we will use the following optimization device where we will optimize the local variables subject to the remaining budget, i.e.,*

$$\max_{y_l^j} \sum_{j=1}^{n_l} b_l^j \log(y_l^j)$$

$$s.t. \sum_{j=1}^{n_l} p_l^{'j} y_l^j = w - \sum_{i=1}^{m_l} p_l^i x_l^i. \tag{33}$$

*We set the Lagrangian of (33) to be*

$$L = \sum_{j=1}^{n_l} b_l^j \log(y_l^j) - \lambda \left[ \sum_{j=1}^{n_l} p_l^{'j} y_l^j + \sum_{i=1}^{m_l} p_l^i x_l^i - w \right]. \tag{34}$$

*From first order conditions of (34) we have*

$$\frac{\partial L}{\partial y_l^j} = \frac{b_l^j}{y_l^j} - \lambda p_l^{'j} = 0 \Rightarrow y_l^j = \frac{\lambda p_l^{'j}}{b_l^j}$$

*and consequently*

$$\frac{y_l^j}{y_l^1} = \frac{p_l^{'1} b_l^j}{p_l^{'j} b_l^1} \Rightarrow y_l^j = \frac{b_l^j \left( w - \sum\limits_{i=1}^{m_l} p_l^i x_l^i \right)}{p_l^{'j} \sum\limits_{j=1}^{n_l} b_l^j}.$$

*From Walras' law we have*

$$p_l^{'1} y_l^1 + \sum_{j=2}^{n_l} p_l^{'j} \cdot \frac{p_l^{'1} b_l^j}{p_l^{'j} b_l^1} y_l^j = w - \sum_{i=1}^{m_l} p_l^i x_l^i \Rightarrow$$

$$y_l^1 \left[ p_l^{'1} \left( 1 + \sum_{j=2}^{n_l} \frac{b_l^j}{b_l^1} \right) \right] = w - \sum_{i=1}^{n_l} p_l^i x_l^i \Rightarrow$$

$$y_l^1 \left[ p_l^{'1} \left( \frac{\sum\limits_{j=2}^{n_l} b_l^j}{b_l^1} \right) \right] = w - \sum_{i=1}^{m_l} p_l^i x_l^i \Rightarrow$$

$$y_l^1 = \frac{b_l^1 \left( w - \sum\limits_{i=1}^{m_l} p_l^i x_l^i \right)}{p_l^{'1} \sum\limits_{j=1}^{n_l} b_l^j}$$

*As a result,*

$$y_l^j = \frac{b_l^j \left( w - \sum\limits_{i=1}^{m_l} p_l^i x_l^i \right)}{p_l^{'j} \sum\limits_{j=1}^{n_l} b_l^j}. \tag{35}$$

*We notice that the derivative of the logarithm of (35) would result to a fraction that would have in the denominator the quantity, $w - \sum\limits_{i=1}^{m_l} p_l^i x_l^i$, and thus a solution of the Marshallian demand would only be possible to be calculated by means of numerical analysis.*

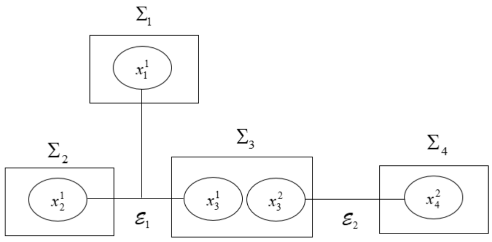


**Fig. 4** Four regions with two inter-regional public goods/services.

**Example 3.** *Assuming that the utility functions of the regions represented in Figure 4 are of the form*

$$F_1 = a_1^1 \log x_1^1 + b_1^1 \log y_1^1$$
$$F_2 = a_2^1 \log x_2^1 + b_2^1 \log y_2^1$$
$$F_3 = a_3^1 \log x_3^1 + a_3^2 \log x_3^2 + b_3^1 \log y_3^1 + b_3^2 \log y_3^2 + b_3^3 \log y_3^3$$
$$F_4 = a_4^1 \log x_4^1 + b_4^1 \log y_4^1$$

*the respective distributed UMP is provided below:*

$$\max_{x_1,y_1,\ldots,x_4,y_4} F_1 + F_2 + F_3 + F_4$$

$$\begin{aligned} s.t. \quad & p_1^1 x_1^1 + p_1^{'1} y_1^1 \leq w_1 \\ & p_2^1 x_2^1 + p_2^{'1} y_2^1 \leq w_2 \\ & p_3^1 x_3^1 + p_3^2 x_3^2 + p_3^{'1} y_3^1 + p_3^{'2} y_3^2 + p_3^{'3} y_3^3 \leq w_3 \\ & p_4^1 x_4^1 + p_4^{'1} y_4^1 \leq w_4 \\ & x = Ez \\ & x_1, y_1, \ldots, x_4, y_4 \geq 0 \end{aligned} \tag{36}$$

*For the general case example the parameters would be,*

*Assuming that the parameters of the first region to be* $a_1^1 = 1.4, b_1^1 = 1$ *for the second region to be* $a_2^1 = 1.5, b_2^1 = 1$*, for the third region to be* $a_3^1 = 1.7, a_3^2 = 1.3, b_3^1 = 0.3, b_3^2 = 0.3, b_3^3 = 0.4$ *and for the for the fourth region to be* $a_4^1 = 1.6, b_4^1 = 1$ *while the respective prices for the first coupling variable are* $p_1 = p_1^1 = p_2^1 = p_3^1 = 2.2$*, for the second coupling variable are* $p_2 = p_3^2 = p_4^1 = 2.1$ *and for the local prices are* $p_1^{'1} = 0.6, p_2^{'1} = 0.5, p_3^{'1} = 0.9, p_3^{'2} = 0.7, p_3^{'3} = 0.8, p_4^{'1} = 0.6$ *with the budgets to be* $w_1 = 22, w_2 = 18, w_3 = 31$ *and* $w_4 = 19$*, the asymptotic values for the coupling variables are*

$$z_1 = x_1^1 = x_2^1 = x_3^1 = 5.415, \quad z_2 = x_3^2 = x_4^2 = 5.214$$

*while the asymptotic values for the local variables are*

$$y_1^1 = 16.814, y_2^1 = 12.176, y_3^1 = 2.713, y_3^2 = 3.488, y_3^3 = 4.07, y_4^1 = 8.95$$

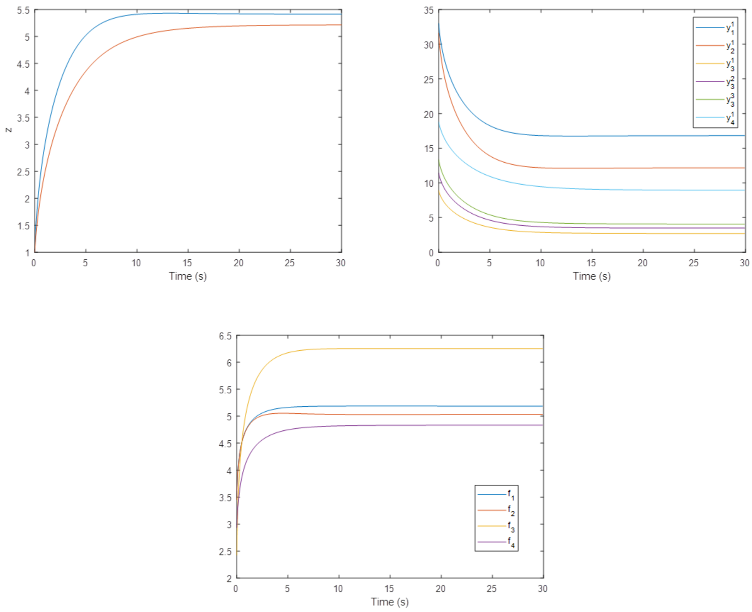


**Fig. 5** (a) Coupling Variables (b) Local Variables (c) Objective Functions

*The asymptotic values of the utility function are*

$$f_1 = 5.187, f_2 = 5.033, f_3 = 6.2538$$

*with the total objective to be* $f = 16.474$. *The convergence processes of the coupling variables, the local variables and the utility functions are presented in Figure 5.*

**Remark 6.** *It is important to note that the policy maker has knowledge of all the values of both the coupling and local variables and is able to understand the prioritization of the public infrastructure from the population viewpoint, something that in real world may affect the policy maker's decisions and these may deviate from the optimal ones that result from the proposed algorithm. Also, a potential extension of the algorithm could be a combinatorial expansion which could be extremely useful in cases where the policy makers are not sure in terms of geographic allocation of the inter-regional infrastructure among regions. In such scenario the policy makers can use the algorithm as many times as the available geographical allocation cases and choose the geographical allocation for which the algorithm results to the best objective value.*

# 5 Conclusion

To sum up, we propose a new approach to the economics of public goods under the decentralized finance setting using distributed optimization techniques in a multi regional geographical environment. In particular, we formulate the problem as a multi-utility optimization problem where each region has its own utility function, and provide a toolkit for planning and for organizing all types of public goods/services, which in turn allows policy makers to better plan the respective policies. The toolkit may also provide other useful policy related insights such as the prioritization of the works across regions and the optimal geographical allocation of public goods/services.

## References


Arrow K. J., Azawa H., Hurwicz L., Uzawa H., Chenery H. B., Johnson S. M., Karlin S. (1958). *Studies in linear and non-linear programming.* Vol. 2. Redwood City CA: Stanford University Press.

Bahl R., Bird R. M. (2018). *Fiscal decentralization and local finance in developing countries*. Cheltenham: Edward Elgar Publishing.

Begg D., Vernasca G., Fisher S., Dornbusch R. (2014). *Economics*. 11th edition. London, Boston MA and other places: McGraw-Hill Publishers.

Berge C. (1973). *Graphs and hypergraphs*. Amsterdam: North-Holland Publishing.

Bergstrom T.C. (1999). Systems of benevolent utility functions. *Journal of Public Economic Theory* 1.1: 71-100.

Bergstrom T.C., Goodman R.P. (1973). Private demands for public goods. *The American Economic Review,* 63.3: 280-296.

Čech P., Tučník P., Bureš V., Husráková M. (2013). Modelling complexity of economic systems with multi-agent systems. *Proceedings of the International Conference on Knowledge Discovery and Information Retrieval and the International Conference on Knowledge Management and Information Sharing.* Vilamoura, Algarve, Portugal, 19 - 22 September 2013. Setúbal: SciTePress, pp. 464–469.

Corchón L., Dahm M. (2011). Welfare maximizing contest success functions when the planner cannot commit. *Journal of Mathematical Economics* 47.3: 309-317.

Debreu G. (1959). *Theory of value: An axiomatic analysis of economic equilibrium*. New Haven CT: Yale University Press.

Debreu G. (1959). Topological methods in cardinal utility theory. *Econometrica, 27*.3: 232–246.

Desmarais-Tremblay M., Johnson M., Sturn R. (2023). From public finance to public economics. *The European Journal of the History of Economic Thought*, 30.5: 934–964.

Evren O. (2014). Scalarization methods and expected multi-utility representations. *Journal of Economic Theory,* 151.C: *30–63.*

Eymann T. (2001). Decentralized economic coordination in multi-agent systems. *Information Age Economy.* Edited by H.U. Buhl, A. Huther, B. Reitwiesner. Heidelberg: Physica, pp. 575–588.

Fedelino A, Ter-Minassian T. (2010). *Making Fiscal Decentralization Work* . IMF Occasional Paper 271.

Gibson, B. (2007). A multi-agent systems approach to microeconomic foundations of macro. University of Massachusetts, Department of Economics Working Paper 2007-10.

Haber, G. (2014). *Economic multi-agent systems: Design, implementation, and application*. Norderstedt: BoD–Books on Demand.

Heintz B., Chandra, A. (2014). Beyond graphs: Toward scalable hypergraph analysis systems. *ACM SIGMETRICS Performance Evaluation Review, 41*.4: 94–97.

Heintz B., Hong R., Singh S., Khandelwal G., Tesdahl C., Chandra, A. (2019). Mesh: A flexible distributed hypergraph processing system. *2019 IEEE International Conference on Cloud Engineering (IC2E 2019) Prague, Czech Republic 24 - 27 June 2019.* Piscataway NJ: Institute of Electrical and Electronics Engineers, pp. 12–22.

Hamel A.H., Wang S.Q. (2017). A set optimization approach to utility maximization under transaction costs. *Decisions in Economics and Finance,* 40.1: 257–275.

Jackson M.O. (2010). *Social and economic networks*. Princeton NJ: Princeton University Press.

Khalil H.K. (2002). *Nonlinear systems*. 3rd edition. Upper Saddle River: Prentice Hall.

King M.A. (1986). A Pigovian rule for the optimum provision of public goods. *Journal of Public Economics,* 30.3: 273-291.

Lancaster K. J. (1966). A new approach to consumer theory. *Journal of Political Economy, 74*.2: 132–157.

Mas-Colell A., Whinston M.D., Green J.R. (1995). *Microeconomic Theory.* Oxford: Oxford University Press.

Oates W.E. (1972). *Fiscal federalism*. New York NY: Harcourt Brace Jovanovich Inc.

Papastaikoudis I., Watson J., Lestas I. (2024). A decentralized control approach in hypergraph distributed optimization decomposition cases. *Applied Network Science, 9,* 58.

Samuelson P.A. (1938). A note on the pure theory of consumer's behavior. *Economica, 5*.17: 61–71.

Samuelson P.A. (1954). The Pure Theory of Public Expenditure. *The Review of Economics and Statistics*, 36.4: 387-389.

Samar S., Boyd S., Gorinevsky D. (2007). Distributed estimation via dual decomposition. *Proceedings of the European Control Conference 2007. Kos, Greece, July 25, 2007. Kos:* Institute of Electrical and Electronics Engineers. pp. 1511–1516.

Tsitsiklis J.N. (1984). *Problems in decentralized decision making and computation*. PhD Thesis. Cambridge MA: Massachusetts Institute of Technology.

Uzawa H. (1960). Preference and rational choice in the theory of consumption. Mathematical Methods in the Social Sciences 1959. Edited by K.J. Arrow, S. Karlin and P. Suppes. Vol. IV. Stanford CA: Stanford University Press, pp. 129-149.

Voloshin V. I. (2009). *Introduction to graph and hypergraph theory*. New York NY: Nova Science Publishers.

Wolf M.M., Klinvex A.M., Dunlavy D. M. (2016). Advantages to Modeling Relational Data using Hypergraphs versus Graphs. *Proceedings of the 2016 IEEE High Performance Extreme Computing Conference. Waltham MA:* Institute of Electrical and Electronics Engineers.

Yang,T., Yi X., Wu J., Yuan Y., Wu D., Meng Z., Hong Y., Wang H., Lin Z., Johansson K. H. (2019). A survey of distributed optimization. *Annual Reviews in Control, 47:* 278–305.

Zhao Q., Zhang Y., Zhang Y., Friedman D. (2017). Multi-product utility maximization for economic recommendation. *Proceedings of the Tenth ACM International Conference on Web Search and Data Mining*. Cambridge: Association for Computing Machinery, pp. 435–443.

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133. P. Prodromídis, S. Petros and A. Petralias, “Analyzing the Unleaded Gasoline Retail Price Patterns in Greece: Apr. 2011 – Dec. 2012”, 2013 (in Greek). Published in *South Eastern Europe Journal of Economics,* vol. 12 (2), 2014, pp. 215-241.
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130. Th. Tsekeris and Kl. Vogiatzoglou, “Regional Specialization and Public Infrastructure Investments: Empirical Evidence from Greece”, 2013. Published in modified form in *Regional Science Policy & Practice*, vol. 6 (3), 2014, pp. 265-289.
129. E. Athanassiou, N. Kanellopoulos, R. Karagiannis, I. Katselidis and A. Kotsi, “Measurement of the Intensity of Regulations in Professions and Economic Activities in Greece via Regulation Indices”, 2013 (in Greek).
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*construction de l'autorité monétaire (XIX$^{e}$-XX$^{e}$ siècle),* Paris: Edition Albin Michel, 2005, pp. 69-86.
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51. A. Balfoussias and V. Stavrinos, "The Greek Military Sector and Macroeconomic Effects of Military Spending in Greece", 1996. Published in N. P. Gleditsch, O. Bjerkholt, A. Cappelen, R. P. Smith and J. P. Dunne (eds.), *In the Peace Dividend*, Amsterdam: North-Holland, 1996, pp. 191-214.
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47. St. Makrydakis, E. Tzavalis and A. Balfoussias, "Policy Regime Changes and the Long-Run Sustainability of Fiscal Policy: An Application to Greece", 1995. Published in *Economic Modelling,* vol. 16 (1), 1999, 71-86.
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3. J. Dutta and H. Polemarchakis, “Credit Constraints and Investment Finance: Evidence from Greece”, 1990. Published in M. Monti (ed.), *Fiscal Policy, Economic Adjustment and Financial Markets,* International Monetary Fund, 1989.
2. L. Athanassiou, “Adjustments to the Gini Coefficient for Measuring Economic Inequality”, 1990.
1. G. Alogoskoufis, “Competitiveness, Wage Rate Adjustment and Macroeconomic Policy in Greece”, 1990 (in Greek). Published in *Applied Economics,* vol. 29, 1997, pp. 1023-1032.