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Adjustments to the Gini Coefficient for Measuring Economic Inequality

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DISCUSSION PAPERS

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CENTRE OF PLANNING AND ECONOMIC RESEARCH

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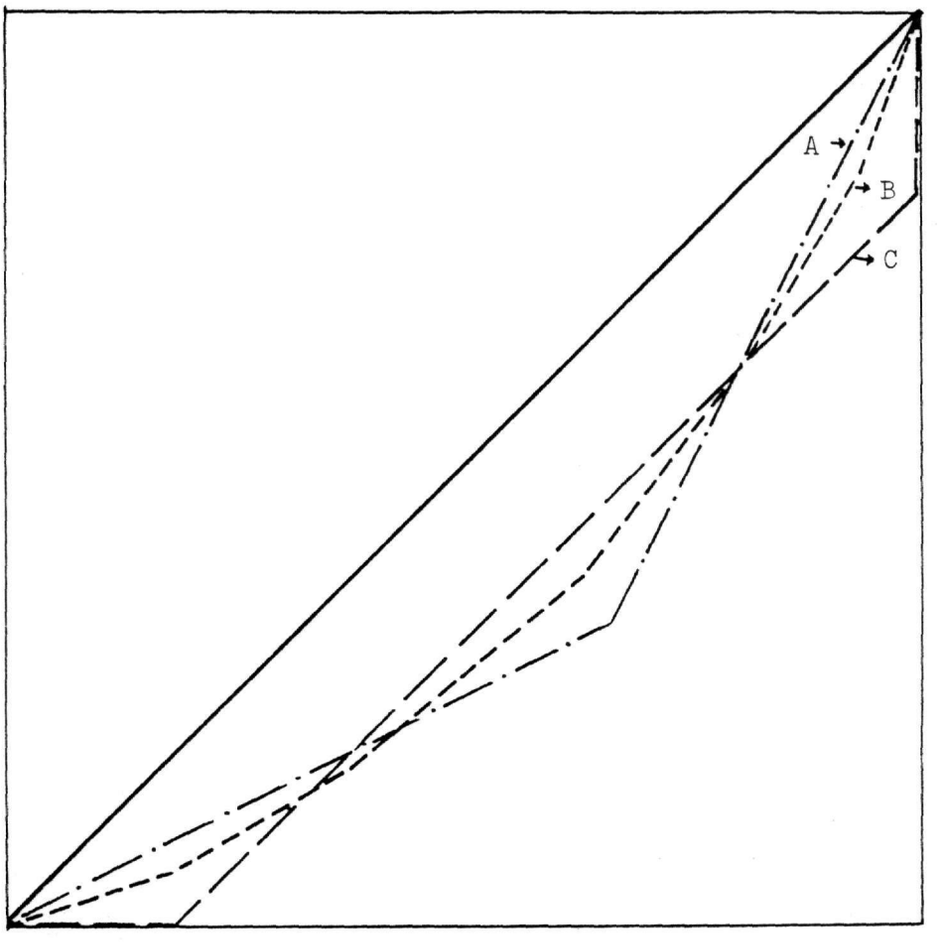
It is well known that income differences of a potentially very different significance as regards equity, economic welfare and economic incentives are treated indifferently in calculating the Gini coefficient. The possible extent of its insensitivity in these respects is illustrated in Diagram 1, where the three Lorenz curves correspond to the same value of the Gini coefficient. The actual figures (curve B) refer to household consumption in Greece in 1982, and the method of calculating the distributions corresponding to Lorenz curves A and C is given in the Appendix.

The distribution regime corresponding to curve C in Diagram 1 is most offensively inequitable, since the bottom rung (18% of the number of households in this instance) has a zero consumption whereas an enormous consumption (18% of total concumption in the present instance) is enjoyed by one immensely rich household. It also seems to be among the poorest as regards economic incentives. The dire condition of the starving bottom rung minimises the possibility of their gaining entrance into the very large lower middle consumption group through increased effort. On the other hand, none of the members of this latter group, which comprises virtually all the gainfully occupied population, stands any real chance of working its way into the one and only baron's position. Only a purely negative incentive may be said to be at work in the form of a fear on behalf of the members of the middle rung lest they sink into the starving bottom. That, however, would probably breed servility and apathy rather than ambition to achieve through effort, and in any case it can hardly come under the concept of economic incentive as a material reward attainable through creativity and increased effort.

DIAGRAM 1



It is usually assumed - and it seems often to be the case - that equity and economic incentives are conflicting objectives, higher marks on the former for any distribution regime implying lower marks on the latter and vice-versa. Given the Gini coefficient in a specific case, however, it becomes clear from what has been pointed out that the distribution regime corresponding to curve C in Diagram 1 combines what is probably one of the most inequitable situations with very poor economic incentive effects, thus getting very low marks on both grounds. In this sense it could serve as a point of reference for the consideration of the equity and economic incentive virtues or vices of different distribution regimes.

It seems to be a good deal more difficult to define a distribution regime which, given the Gini coefficient, would combine high marks on both the equity and the economic incentive grounds. When one has eliminated all the inequality which seems to serve no very useful purpose for economic incentive - and there may well be a lot of it in many cases - there may be no way of obtaining any further gains in equity without weakening economic incentive.

As a counterpoint to the distribution regime corresponding to Lorenz curve C in Diagram 1, however, one may consider that which corresponds to curve A in the same diagram. The latter would seem to get very high marks since both poverty and riches are eliminated. Households are divided into two groups only, namely that with the lower middle consumption and that with the higher middle one. With only one degree of differentiation and given the Gini coefficient, it is very difficult to see how one could improve on the

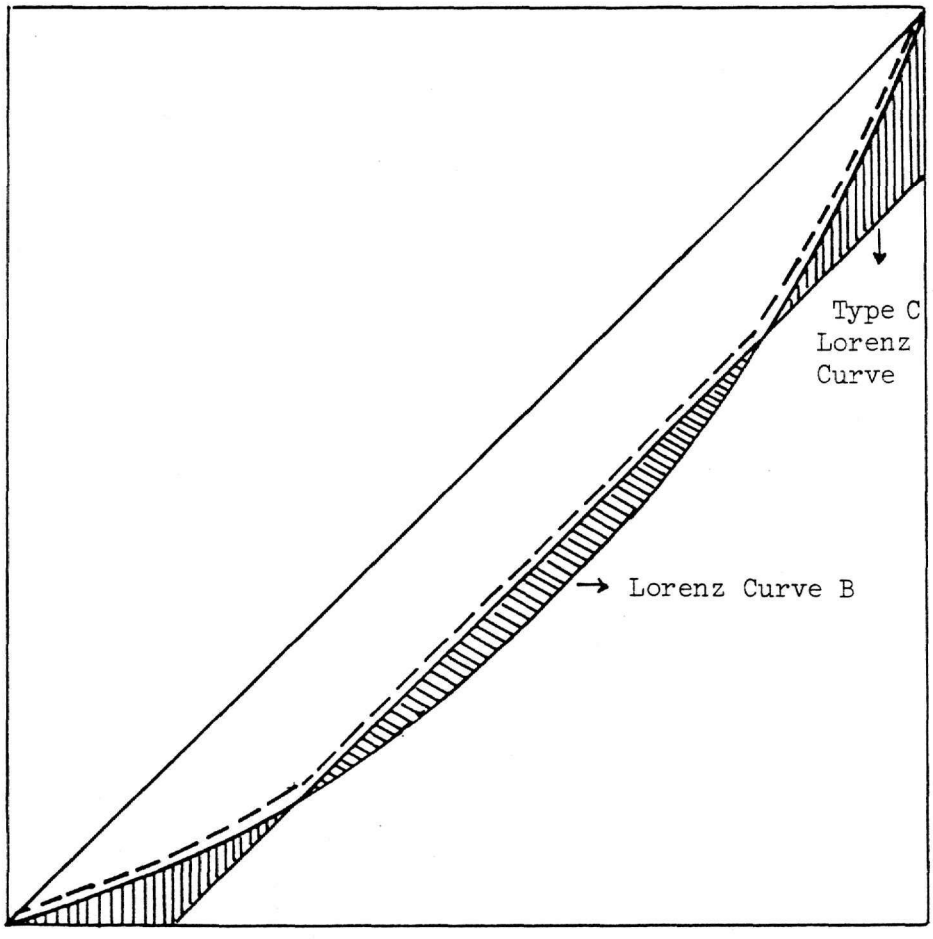
equity situation. Under the circumstances, any alternative distribution arrangement would increase the gap between the richer and the poorer.

It can also easily be seen that the inequalities allowed within the regime in question function very well from the point of view of economic incentive. It seems fair to suggest that the effectiveness of a given inequality feature of a distribution regime in eliciting effort and economic creativity is enhanced when the potential reward in a given instance seems worthwhile and the extra effort required every time for achieving it seems well within reach, involving no more than a calculated risk. When the potential reward is poor in relation to the likely effort required, or when the odds of success are long, the corresponding incentive weakens. One is left with inequalities that are not very productive in terms of economic incentive. In the present case, the higher middle consumption is a fairly visible objective of economic ambition for the more populous group with the lower middle consumption level, and the probable amount of extra effort required to upgrade oneself in this respect may seem to a lot of people to be sufficiently within reach.

Generally it can be said that in the context of a given Gini coefficient the distribution regimes corresponding to Lorenz curves A and C seem to be the analogues of complete inequality and complete equality respectively.

Now consider the shaded areas in Diagram 2, where the Gini coefficient corresponding to both Lorenz curves is equal. It follows that the densely shaded area is equal to the lightly shaded one. The boldly shaded

DIAGRAM 2



area on the other hand is equivalent to the difference between the mean difference, Δ, corresponding to the actual Lorenz curve indicated by the letter B in Diagram 2 and the mean difference corresponding to the broken Lorenz curve in the Diagram, Δ_C. Hence one can use the mean difference formula to calculate the shaded area in Diagram 2 as a fraction of the total area under the diagonal, thus rendering it comparable to the Gini coefficient. In what follows the shaded area of the kind shown in Diagram 2 as a fraction of the total area under the diagonal will be denoted by g. In symbols $g = \frac{\Delta - \Delta_C}{N^2 - \mu}$ N stands for the total frequency and μ for the average quantity distributed. The letter γ will be employed for a similar purpose when the shaded area arises in comparing an actual Lorenz curve with a type A curve rather than with a type C one (Diagram 1).

Bearing in mind these definitions, consider two distribution regimes 1 and 2, where the values of the Gini coefficient, G_1 and G_2 are equal while $g_1 \neq g_2$. One could simply take this as a warning that the superficial equivalence of the two regimes in question, indicated by the equality of the two Gini coefficients, conceals important differences. It would be possible, however, to proceed one step further by determining adjusted values for the Gini coefficient, taking into account the equity and economic incentive attributes of the distribution regimes under comparison, as these may be reflected in the value of g. Such adjusted coefficients might afford a more satisfactory basis for comparison than the Gini coefficient.

Thus if one has a given type C Lorenz curve as a point of reference, as is the case in the previous example when $G_1 = G_2$, one might be interested

in the adjustment to the initial values of G that would be necessary, so that $g_1 = g_2$. Such an adjustment would yield comparable coefficients as regards the equity and economic incentive attributes of the corresponding distributions in the sence that the distance of the respective Lorenz curves from a fixed type C reference curve would be the same.

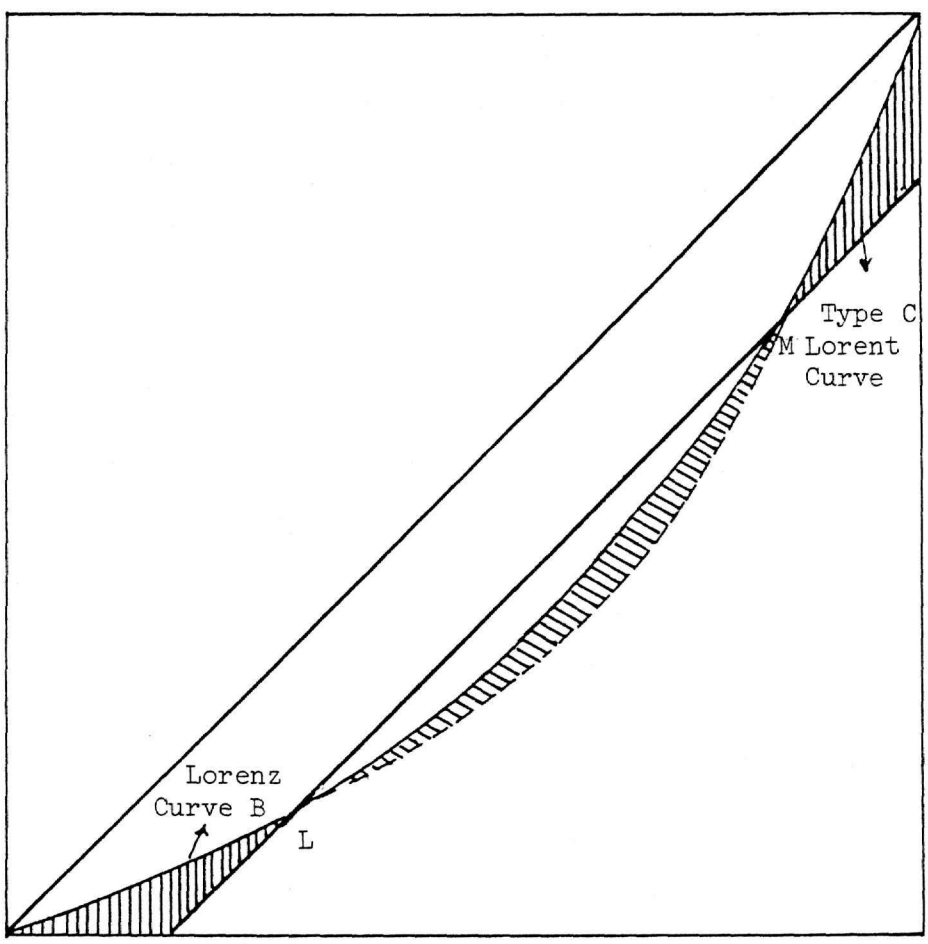
In principle one could use G_1 or G_2 as point of departure and one could equate the two values of g to either g_1 or g_2. It is suggested, however, that in most cases it may be found more appropriate to adjust the Gini coefficients in such a way as to equate the values of g to the higher of the two and take the corresponding Gini coefficient as the point of departure.

As is clear from the above, any feature of a distribution regime which, given G, increases g will most probably be desirable on equity grounds since it would amount to an improvement of the lot of the poor and would reduce the degree of concentration of the aggregate distributed in the hands of the rich. Within limits, such a feature would also be favourable for incentives since it would imply an increasing concentration of the inequality admitted within a range that increases its effectiveness as a potential reward for increased effort and economic creativity. Doubts as regards the favourable character of an increased g, given G, could be justified only when the value of γ becomes too small, since in such a case economic incentives might be dangerously weakened. Therefore, as long as the value of γ continues to be at least a sizeable fraction of g, signifying a fairly large number of substantial steps in the ladder from

the lowest to the highest values of the aggregate distributed, it may reasonably be assumed, at least as a first approximation, that a higher value of g is preferable to a lower one, other things being equal. Incidentally, as a fairly wide sample of actual cases indicates, γ tends to be smaller but not much smaller than g.

Assume that in the above case $g_1 = 0.08$ and $g_2 = 0.05$. In view of the above distribution regime 1 may be considered as the more advantageous and the adjusted value of G_1, $\bar{G}_1$ would be equal to the initial value. It can be shown that the smallest increase in the value of G_2 necessary in order to yield $g_2 = 0.08$ with reference to the common type C Lorenz curve for both regimes would be equal to $(g_1 - g_2)$. An adjustment yielding $\bar{G}_2 = G_2 + (g_1 - g_2)$ is illustrated in Diagram 3, where the shaded area yields such a difference in the mean differences corresponding to the initial and the adjusted distributions as to increase g_2 from 0.05 to 0.08. It is obvious that only when the shift from the continuous to the dented Lorenz curve is all concentrated between the points of intersection L and M does the increase in inequality implied in the shift affect G and g to the same extent. Arrangements taking the dented Lorenz curve within the dotted areas would inevitably require an increase in G greater than the increase in g. Thus in our example we would have the adjusted values of the Gini coefficient $\bar{G}_1 = 0.30$ and $\bar{G}_2 = 0.33$ where $\bar{G}_2$ has been derived by the smallest possible increase in the mean difference of distribution regime 2, which could render g_1 equal to g_2. The relationship between these adjusted coefficients would incorporate not only the mean differences

DIAGRAM 3



of the two distributions, but also an impact of their equity and economic incentive attributes with reference to a common type C Lorenz curve.

In the general case the adjustment formula would be:

$$\bar{G}_i = G_i + \left(g_{max} - g_i \frac{G_{max}}{G_i}\right)$$

where $i = 1,2,\ldots n$, and g_{max} and G_{max} the highest value of g and G in the corresponding sets. This formula can be reduced to that used in the above example when $G_i = G_{max}$.

It is virtually impossible to devise any scalar measure of inequality which will be sensitive to every aspect of a distribution regime, thus establishing a one to one correspondence between the value of the index and the identity of the distribution regime. And even if that were possible, no credible efficiency or equity ranking criteria could be established. Thus g and γ suffer from a degree of insensitivity to important differences in distribution regimes and the same coefficient may conceal different realities. If, however, one accepts the assumption that a connection can be established between the locus of economic differences - as distinguished from the overall mean difference - and the corresponding equity and economic incentive effects along the lines suggested above, then g in conjunction with γ can add very considerably to the degree of discrimination of the standard measure. These coefficients provide one with a means to get a view, albeit a dim one, of what lies behind a global mean difference, thus reducing the extent to which widely different distribution regimes may be

treated indifferently as may happen when one relies exclusively on the Gini coefficient.

In comparing any number of Gini coefficients and calculating the values of the $\bar{G}_i$'s, the g_i's and the γ_i's, one may encounter essentially two kinds of situations. Either the adjusted values of the Gini coefficients will be considerably different from the unadjusted ones yielding a different ranking, or they will not. In the latter case one would be justified in drawing conclusions from the comparison of the G values with a bit more confidence than that which would be warranted on the basis of the values of G alone. In the former case the conclusions from the $\bar{G}$ values may be different from those derived on the basis of the G values. In such a case it would be unwise to trust the $\bar{G}$ values mechanistically. If, however, a closer examination of the patterns of distribution involved tends to confirm the significance of the findings, one might choose to rely on the $\bar{G}$ values rather than the G ones.

It is beyond the scope of this paper to calculate $\bar{G}$ values for a wide variety of distributions and it is quite clear that comparisons of $\bar{G}$ coefficients belonging to widely disparate situations - or indeed G coefficients for that matter - might be devoid of much significance. It is, however, useful by way of example to present the results for one of the exercises carried out in an effort to appreciate the singificance of the above.

I considered it advisable to use household expenditure survey data rather than income or earnings data which constitute the usual basis for gauging economic inequality, since the former tend to be statistically more

Household Consumption Size Distribution
Gini Coefficients and g and γ Coefficients

Country	Gini coefficient	g	γ
Canada (1974)	0.2965	0.0751	0.0497
France (1963/1964)	0.2150	0.0557	0.0452
Greece (1974)	0.3611	0.0796	0.0673
(1982)	0.3255	0.0807	0.0585
Ireland (1979)	0.2916	0.0718	0.0612
Portugal (1973/1974)	0.3685	0.0873	0.0652
Spain (1980/1981)	0.2634	0.0682	0.0523
Sweden (1978)	0.1898	0.0520	0.0427
United Kingdom (1983)	0.2758	0.0721	0.0519

Source: The household consumption expenditure data utilised have been obtained from the following sources: Canada, Urban Family Expenditure 1974, Ministry of Industry, Trade and Commerce, Ottawa. France, Enquête sur les Budgets Familiaux, 1963-1964, Paris, Imprimerie Nationale. Greece, Household Expenditure Survey Results, 1974 and 1982, Athens, The National Statistical Service. Ireland, Household Budget Survey, Annual Urban Inquiry Results, 1979, The Stationery Office. Portugal, Inquerito ad Despesas Familiares 1973-1974, Instituto Nacional de Estatistica. Spain, Encuesta de Presupuestos Familiares, 1980-1981, Vol. I, Instituto Nacional de Estadistica. Sweden, The Family Expenditure Survey, 1978, Part I, National Central Bureau of Statistics. United Kingdom, Family Expenditure Survey, 1983, The Government Statistical Service.

reliable and relate directly to the standard of living. Furthermore, household expenditure data incorporate the optimising decisions of individuals and households regarding the standard of living which can be secured on the basis of the expected lifelong income. To the extent that individuals or households equate lifelong consumption with lifelong income in a situation of complete lifelong income equality, the differences in consumption levels at different stages of the life cycle and the measured degree of inequality on the basis of standard methods would reflect such optimising decisions, thus establishing a one to one correspondence between lifelong income inequality and the observable degree of inequality in consumption at a point of time. Other things being equal, any increase in lifelong income inequality would be reflected in a higher degree of consumption inequality than that which would correspond to the optimising time pattern of consumption of equal lifelong incomes. Since the value of any inequality coefficient has no particular significance beyond its comparative difference from another, this might be a less objectionable approach[1] for taking into account the impact of income variations as a result of differences in age than the Paglin correction to the Gini coefficient[2].

As can be seen from the above table, the calculation of g and γ tends by and large to confirm rather than question the picture emerging from the G values. There seem to be important exceptions however. Thus, considering the two figures for Greece, the reduction in inequality after taking into account g and obtaining $\bar{G}$ values is approximately 25% bigger than what is the case when one compares the G values. In assessing the

impact of measures to reduce inequality, or in examining developments in distribution, such a finding would certainly merit examination. It is also revealed that Sweden not only has the lowest G and consequently a lower overall degree of inequality, but also the highest relative distance both from the type C and the type A curves (the fraction $\{(g+\gamma)/G\}$ and g/G is the highest). This means that distribution policies there have managed to improve on equity by reducing both poverty and riches and at the same time preserve a relatively higher degree of income differentiation concentrated to a grater extent in the range where it may be expected to function better as an incentive (away from both extremes and toward the middle ranges).

APPENDIX

Let f_i where i = 1,2,3,... n indicate the ith recipient and x_i indicate the corresponding value of the distributed variable received (income etc.). Given the value of the Gini coefficient, G, in a specific case, it can easily be shown that the distribution corresponding to the curves of type C and A as per Diagram 1 respectively is as follows:

Type C Curve		Type A Curve	
number of recipients	value of distributed variable	number of recipients	value of distributed variable
$(1-\sqrt{1-G})\ \Sigma f_i$	0	$\frac{1+G}{2} \cdot \Sigma f_i$	$\frac{(1-G)\ \Sigma f_i x_i}{(1+G)\ \Sigma f_i}$
$\sqrt{1-G} \cdot \Sigma f_i - 1$	$\frac{\Sigma f_i x_i}{\Sigma f_i}$	$\frac{1-G}{2} \cdot \Sigma f_i$	$\frac{(1+G)\ \Sigma f_i x_i}{(1-G)\ \Sigma f_i}$
1	$(1-\sqrt{1-G})\ \Sigma f_i x_i$		

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2. M. Paglin cited in (1) above.

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